

# PHILOSOPHIES OF MATHEMATICS IN DIALOGUE

PRINCETON UNIVERSITY  
AARON BURR HALL, 219

20-21 MAY 2023

## SCHEDULE

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### SATURDAY, 20 MAY 2023

9:00 - 9:15 coffee

Chair: Janet Folina

9:15 - 10:15 – Øystein Linnebo, “Definiteness: Between Mathematical Practice and Philosophical Analysis”

10:30 -11:30 – Silvia De Toffoli, “A Fallibilist Account of Mathematical Justification”

11:45 -12:45 – Colin McLarty, “Let's Talk About *Some* Mathematics”

1:00 – 2:30 lunch

Chair: Bill D'Alessandro

2:30 - 3:30 – Patricia Blanchette, “Conceptual Analysis in Logic and Mathematics”

3:45 - 4:45 – José Ferreirós, “A Conceptualist Take on Structuralism”

5:00-6:30 – Round table on different perspectives in the philosophy of mathematics (with John Burgess, Jessica Carter, Michael Harris, and Keith Weber)

Conference Dinner

### SUNDAY, 21 MAY 2023

9:00 - 9:15 coffee

Chair: John Burgess

9:15 - 10:15 – Harty Field, “Extreme Pluralism About Mathematics and Logic”

10:30 -11:30 – Alan Baker, “Applied Mathematics in Practice: A Tale of Two Graphs”

11:45 -12:45 – Stewart Shapiro, “Potentiality in Mathematics”

1:00 – 2:30 lunch

## ABSTRACTS

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### **Alan Baker, "Applied Mathematics in Practice: A Tale of Two Graphs"**

In this talk, I examine two examples of the application of graph theory to physical phenomena. The first example, involving the bridges of Königsberg, is well-known to philosophers. The second example, involving the structure of RNA molecules, is not. In both cases, my interest is in the nature of the relation between the mathematical graph and the physical phenomenon. In particular, does the graph have some special, non-arbitrary relation to the structure of the phenomenon? I then go on to discuss a second way to map phenomena onto graphs, using dual graphs, which has important implications for the above question. Finally, I raise and defend the suggestion that a more productive way to think about applications of this sort is as relating not to physical phenomena per se but to game-like problem scenarios.

### **Patricia Blanchette, "Conceptual Analysis in Logic and Mathematics"**

One of the most important capabilities provided by modern logic to the art of mathematics is the ability to demonstrate consistency and independence results. The paradigm example here is the field of non-Euclidean geometries. The modern technique for proving independence – the technique of providing a set-theoretic model of a theory expressed in a language of formal logic – requires a certain sensitivity to the way in which the mathematical theory is expressed in that formal language, which in turn requires a sensitivity to the mathematical concepts and relations involved, and to their analysis into simpler concepts and relations. This talk investigates this dependence between conceptual analysis and consistency/independence results, in a way that I hope will clarify some of the more general ways in which formal logic and conceptual analysis work hand-in-hand, especially in the application of logical techniques to mathematics.

### **Silvia De Toffoli, "A Fallibilist Account of Mathematical Justification"**

According to a widely held view in the philosophy of mathematics, direct inferential justification for mathematical propositions (that are not axioms) requires proof. I challenge this view. I argue that certain fallacious mathematical arguments considered by the relevant subjects to be correct can confer mathematical justification. But mathematical justification doesn't come for cheap: not just any argument will do. I suggest that, in order to be successful in transmitting justification, an argument must satisfy specific standards, some of which are social. The picture I delineate is a hybrid form of phenomenal conservatism. Although in this talk I focus on mathematical inferential beliefs, the view on offer generalizes straightforwardly to other inferential beliefs.

### **José Ferreirós, “A Conceptualist Take on Structuralism ”**

This paper defends a conceptualistic version of structuralism as the most convincing way of elaborating a philosophical understanding of structuralism. The tradition of “conceptual mathematics” in the period 1850 to 1940 (Riemann, Dedekind, Hilbert, E. Noether) led to a structuralist methodology in pure mathematics. But there is a tension between the ‘presuppositionless’ approach of those authors, and the platonism of some recent philosophical versions of structuralism. I argue that one can resolve this tension, admitting ‘logical objects’ understood in minimalist terms, interpreted from a semiotic point of view, and introducing the basic tenets of conceptual structuralism. The paper is devoted to an open discussion of the assumptions behind conceptual structuralism, including arguments to show that the objectivity of mathematics can be explained from the adopted standpoint – without denying that advanced mathematics builds on hypothetical assumptions (Riemann, Peirce, Hilbert).

### **Harty Field, “Extreme Pluralism About Mathematics and Logic”**

I’ll discuss various different ways of trying to elucidate an extreme pluralism about math and about parts of logic, while suggesting that the extension to all of logic is problematic. One way to elucidate the pluralism that might seem to get around this is conventionalism, but I’ll argue both that it has trouble differentiating itself from the other elucidations and that its ability to extend to all of logic is doubtful.

### **Øystein Linnebo, “Definiteness: Between Mathematical Practice and Philosophical Analysis” (incorporating joint work with Laura Crosilla)**

Notions of definiteness play little role in today’s mainstream mathematical practice. This makes it tempting to dismiss philosophers’ appeals to such notions as “detached philosophy” and “mathematically sterile”. To reject this criticism, we document how some of the most prominent mathematicians during the first half century of Cantorian set theory (1880s though 1930s) appealed to notions of definiteness in their own theorizing. We distinguish two main notions of definiteness: *intensional* definiteness, i.e. that a concept is well-defined for any individual object to which it might be applied, and *extensional* definiteness, i.e. that the concept has a well-defined extension, namely, a collection of all the objects to which it applies. Both notions admit of interesting philosophical and logical analyses, which in turn give rise to valuable mathematics. These analyses involve modal or intuitionistic logic (or both).

### **Colin McLarty, "Let's Talk About *Some* Mathematics"**

By "some" mathematics I mean not mathematics according to one or another general definition but mathematics being done, now, over in the math department. Or done at some other specific place, or historic time, by specified mathematicians. Dialogue between philosophies of mathematics can mean philosophers talking with each other about each

other's ideas, and that's great. There should be more of that than there is. But the dialogue is likely to run longer and go deeper the more it draws on specific (relatively) uncontroversial productive mathematics as (potentially) common ground for discussion.

### **Steward Shapiro, "Potentiality in Mathematics"**

The purpose of this talk is to explore the use of modal logic and/or intuitionistic logic to explicate potentiality and incomplete or indeterminate domains in mathematics. Our primary applications are the traditional notion of potential infinity, predicativity, a version of real analysis based on Brouwerian choice sequences, and a potentialist account of the iterative hierarchy in set theory. We present a unified framework in which these phenomena can be described and studied. We then locate various views—historical as well as contemporary (including some developed by ourselves)—in this framework.