1. Aristotle’s thesis in Posterior Analytics 1.26

In the opening sentence of the Analytics, Aristotle announces that the subject of the treatise is demonstration (ἀπόδειξις). A demonstration, for Aristotle, is a kind of deductive argument. Specifically, a demonstration is a deduction through which, when we possess it, we have scientific knowledge (ἐπιστήμη). While the Prior Analytics is a study of deduction in general, the Posterior Analytics examines the nature of demonstration.

In Posterior Analytics 1.24–6, Aristotle compares different kinds of demonstration. He begins by explaining why, in his view, universal demonstrations are better than particular
ones, and positive demonstrations are better than negative ones (1.24–5). He goes on, in chapter 1.26, to argue that direct demonstrations are better than those that employ the method of reductio ad impossibile. In the first sentence of the chapter, Aristotle writes:

'Επεί δ' ἡ κατηγορικὴ τῆς στερητικῆς βελτίων, δήλου ὅτι καὶ τῆς εἰς τὸ ἀδύνατον ἀγούσης. (Post. An. 1.26 87a1–2)

Since positive demonstration is better than privative demonstration, clearly it is also better than that which leads to the impossible.

In this sentence, Aristotle is referring to main result of the preceding chapter, that direct positive demonstrations are better than direct negative (or, privative) ones. Based on this, he seeks to establish in chapter 1.26 that direct positive demonstrations are better than those by reductio ad impossibile. He does so by arguing that direct negative demonstrations are better than those by reductio. In particular, he argues that direct negative demonstrations are superior in explanatory power to those by reductio in that they proceed from premises which are prior to the conclusion. In the final section of chapter 1.26, Aristotle states this thesis as follows:

ei οὖν ἡ ἐκ γνωριμωτέρων καὶ προτέρων κρείττων, εἰσὶ δ' ἀμφότεραι ἐκ τοῦ μὴ εἶναι τι πιστά, ἀλλ' ἡ μὲν ἐκ προτέρου ἢ δ' ἐξ υστέρου, βελτίων ἀπλώς ἀν εἶ ὑπὲρ τῆς εἰς τὸ ἀδύνατον ἡ στερητικὴ ἀπόδειξις, ὥστε καὶ ἡ ταύτης βελτίων ἡ κατηγορικὴ δήλον ὅτι καὶ τῆς εἰς τὸ ἀδύνατον ἐστὶ βελτίων. (Post. An. 1.26 87a25–30)

Thus, if a demonstration which proceeds from what is more known and prior is superior, and if in both kinds of demonstration conviction proceeds from something’s not being the case, but in the one from something prior and in the other from something posterior, then privative demonstration will be better without qualification than demonstration leading to the impossible. Consequently, it is also clear that positive demonstration, which is better than privative demonstration, is better than that which leads to the impossible.

According to this passage, both direct negative demonstrations and those by reductio make use of negative propositions, that is, propositions asserting that ‘something is not the case’. They differ from each other with respect to the priority relations obtain between their premises and the conclusion. Direct negative demonstrations proceed from premises which are more known than and prior to the conclusion, whereas demonstrations by reductio proceed from premises which are posterior to the conclusion. In Posterior Analytics 1.2, Aristotle distinguishes two kinds of priority: priority ‘in nature’ and priority ‘to us’. Likewise, he distinguishes between being more known ‘in nature’ and being more known ‘to us’. In chapter 1.26, Aristotle makes it clear that the sense in which the premises of direct negative demonstrations are prior to the conclusion is priority in nature (φύσει, 87a17). Accordingly,

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2 Post. An. 1.2 71b33–72a5. In this passage, the distinction between being more known in nature and being more known to us is treated as equivalent to the distinction between priority in nature and priority to us; see McKirahan 1992: 30–1, Barnes 1993: 95–7.
when Aristotle refers to these premises as ‘more known’, he does not mean that they are
necessarily more known to us, but that they are more known in nature.3

By contrast, the premises of demonstrations by *reductio* are not prior but posterior in
nature to the conclusion. In Aristotle’s view, premises that are not prior in nature to the
conclusion fail to reveal the cause (αἰτία) of the *demonstrandum*, and hence are not
explanatory of the conclusion (αἰτία τοῦ συμπεράσματος, 1.2 71b22).4 Thus, the premises of
demonstrations by *reductio* are not explanatory of the conclusion. At the same time, Aristotle
is clear that, in order to have scientific knowledge of a thing, one needs to grasp the cause, or
explanation, of that thing (1.2 71b9–12). For this reason, demonstrations by *reductio* are not
capable of producing scientific knowledge of the *demonstrandum*. This does not, of course,
mean that Aristotle denies the validity of these demonstrations. There is no doubt that he
regards them as valid deductive arguments capable of producing conviction (πίστις) of the
*demonstrandum*.5 He denies, however, that they can produce scientific knowledge (ἐπιστήμη).

In *Posterior Analytics* 1.2, Aristotle defines a demonstration as a deduction that is
capable of producing scientific knowledge (71b17–19). Accordingly, he requires that the

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4 Aristotle maintains that, in order for the premises of a demonstration to be explanatory (αἰτία) of the
conclusion, they must be prior in nature to the conclusion (1.2 71b31); see Zabarella 1608: 660c–d, Bronstein
2016: 128. For the premises to be explanatory of the conclusion is for them to reveal the cause (αἰτία) of the
*demonstrandum* (1.2 71b30–1 in conjunction with 71b9–12); cf. Bronstein 2016: 35–8.

5 This is clear from the fact that Aristotle regards demonstrations by *reductio*, just like direct negative
demonstrations, as ‘convincing’ (πιστάται, 87a26); cf. Philoponus in *Post. An.* 298.28–299.2.
premises of a demonstration be more known in nature than, prior in nature to, and
explanatory of the conclusion (71b19–25). Since demonstrations by reductio fail to meet
these conditions, they are not genuine demonstrations as defined in chapter 1.2. Instead, they
are demonstrations in a broader sense. Aristotle countenances such a broader sense of
‘demonstration’, for example, in Posterior Analytics 1.13, when he distinguishes deductions
‘of the why’ (τοῦ διότι) from deductions ‘of the fact’ (τοῦ ὅτι). Although deductions ‘of the
fact’ are not explanatory and do not reveal the cause of the demonstrandum, Aristotle is
willing to refer to them as ‘demonstrations’ (78a30, 78b14). Similarly, when Aristotle speaks
of ‘demonstrations’ by reductio ad impossibile in 1.26, these may be demonstrations ‘of the
fact’, but they are not genuine demonstrations. By contrast, direct negative demonstrations
proceed from premises which are prior in nature to the conclusion. Thus, provided that they
satisfy the other conditions laid down in Aristotle’s characterization of demonstration in
chapter 1.2, these are genuine demonstrations in which the premises are explanatory of the
conclusion. As such, they are capable of producing scientific knowledge.

Aristotle’s view that demonstrations by reductio fail to be explanatory proved
influential over the centuries. For example, the view is endorsed by Proclus in his
commentary on the first book of Euclid’s Elements:

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*At 71b21–2, ‘more known’ and ‘prior’ are used to mean ‘more known in nature’ and ‘prior in nature’; see
ὅταν μὲν οὖν ὁ συλλογισμὸς ἐδίδασκεν τοῖς γεωμέτραις, ἀγαπῶσι τὸ σύμπτωμα μόνον εὑρεῖν, ὅταν δὲ διὰ προηγουμένης ἀποδείξεως, τότε πάλιν, εἰ μὲν ἐπὶ μέρους αἱ ἀποδείξεις γίγνοντο, οὔπω δῆλον τὸ αἰτίον, εἰ δὲ καθ’ ὅλον καὶ ἐπὶ πάντων τῶν ὁμοίων, εὐθὺς καὶ τὸ διὰ τί γίγνεται καταφανές. (Proclus in Eucl. I 202.19–25)

When geometers reason through the impossible, they are content merely to discover the property [of a given subject]. But when their reasoning proceeds through a principal demonstration, then, if such demonstration is partial, the cause is not yet clear, but if it is universal and applies to all like things, the ‘why’ is at once made evident.7

According to Proclus, demonstrations by reductio ad impossibile serve to establish the fact that a given subject has a certain property. They do not, however, reveal the cause, or the ‘why’, of that fact. Instead, the cause can be revealed by means of a ‘principal’ demonstration, that is, a direct demonstration.8 Provided that the direct demonstration exhibits the appropriate level of generality, it succeeds in revealing the cause of the demonstrandum. Thus, like Aristotle, Proclus holds that direct demonstrations possess an explanatory power that those by reductio lack. Proclus was thoroughly familiar with Aristotle’s logical works and wrote a commentary on the Posterior Analytics.9 His commentary on the Elements contains

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7 The translation follows the one given by Heath (1908: 150 n. 1), with some modifications.

8 As Heath points out, the phrase ‘principal demonstration’ (προηγουμένη ἀπόδειξις) at 202.21–2 is used by Proclus to refer to direct demonstrations (Heath 1908: 150 n. 1; pace Morrow 1992: 158–9). For a similar use of the phrase, see Proclus in Eucl. I 321.16–20.

9 As a student, Proclus learned Aristotle’s logical writings by heart (Marinus Vita Procli 9.34–6 Saffrey, Segonds). For Proclus’ (lost) commentary on the Posterior Analytics, see Helmig 2010: 27–9.
numerous references to the Posterior Analytics, including an exposition of Aristotle’s account of exactness in chapter 1.27. Thus, it seems clear that, in his remarks on demonstration by reductio, Proclus follows Aristotle’s treatment in chapter 1.26.

The same view of demonstration by reductio was endorsed by a number of thinkers in the early modern period. For example, in his 1615 essay De mathematicarum natura dissertatio, Giuseppe Biancani denied that demonstrations by reductio proceed ‘from a cause’, citing the passage from Proclus’ commentary just quoted. In his Lectiones mathematicae from the 1660s, Isaac Barrow asserts that, as for reasoning by reductio ad impossibile, ‘Aristotle teaches, and everyone grants, that reasoning of this sort does not at all furnish knowledge that is very perspicuous and pleasing to the mind’. The point is made more explicit by Arnauld and Nicole in the Port-Royal Logic:

Those demonstrations which show that a thing is such, not by its principles, but by some absurdity which would follow if it were not so, are very common in Euclid. It is clear, however, that while they may convince the mind, they do not enlighten it, which ought to be the chief result of knowledge; for our mind is not satisfied unless it knows

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10 For Proclus’ exposition of Post. An. 1.27, see in Eucl. I 59.10–60.1; cf. Morrow 1992: 47.


12 Biancani 1615: 10, 12, and 20.

13 Barrow 1860: 377. Accordingly, Barrow maintains that reasoning by reductio, ‘as all acknowledge, is very obscure and ignoble’ (1860: 357).
not only that a thing is, but why it is, which cannot be learnt from a demonstration by reduction to the impossible. (Arnauld & Nicole, *Port-Royal Logic* IV 9.3)

Arnauld and Nicole criticize Euclid and other geometers for giving demonstrations by *reductio* in cases when a direct demonstration is available. These geometers, they argue, ‘have not sufficiently observed, that it does not suffice for the establishment of a perfect knowledge of any truth to be convinced that it is true, unless, beyond this, we penetrate into the reasons, derived from the nature of the thing itself, why it is true’ (IV 9.1).

A similar view is expressed by Kant in the *Critique of Pure Reason*. In the course of specifying the methods of proof admissible in the discipline of pure reason, Kant excludes proof by *reductio ad impossibile*, or, ‘apagogic’ proof, on the following grounds:

The third special rule of pure reason, if it is subjected to a discipline in regard to transcendental proofs, is that its proofs must never be apagogic but always ostensive. The direct or ostensive proof, in all kinds of cognition, is that which combines with the conviction of truth insight into the sources of the truth; the apagogic proof, by contrast, can produce certainty, but cannot enable us to comprehend the truth in its connection with the grounds of its possibility. Hence the latter is more of an emergency aid than a procedure which satisfies all the aims of reason. (Kant, *Critique of Pure Reason* A 789 / B 817)
In the 19th century, this attitude to ‘apagogic’ proofs was adopted by a number of theorists working in the Kantian tradition. In his *Theory of Science*, Bolzano argues that proofs by *reductio*, while they can produce conviction, cannot exhibit ‘the objective ground’ of the *demonstrandum*. Similarly, Trendelenburg states in his *Logical Investigations* that ‘indirect proof, as Aristotle has already shown, possesses less scientific value than direct proof. … Indirect proof does not provide any insight into the inner grounds of the thing’. This is not to say that all these theorists agree on why demonstrations by *reductio ad impossibile* fail to be explanatory. They may adopt different ways of answering this question. Nonetheless, they all share the same view, which originates with Aristotle’s discussion in *Posterior Analytics* 1.26.

While Aristotle’s thesis in *Posterior Analytics* 1.26 was widely influential, the precise import of the thesis has remained somewhat obscure. There is no consensus among scholars on why demonstrations by *reductio* fail to be explanatory in Aristotle’s view. This is mainly because the argument Aristotle gives for his thesis in *Posterior Analytics* 1.26 is compressed and raises a number of interpretive questions. In fact, the argument has been deemed problematic since antiquity. In his commentary on 1.26, Philoponus reports that ‘all commentators together (ἂπαξάπαντες οἱ ἐξηγηταί) have attacked Aristotle on the exposition of these things, saying that he gives an incorrect account of deduction through the

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15 Bolzano 1837: IV §530, pp. 270–1; see also pp. 277–8.

16 Trendelenburg 1870: 440. For his discussion of *Posterior Analytics* 1.26, see Trendelenburg 1862: 127–9.
impossible’. Similarly, Zabarella notes that ‘I have considered this passage for a very long time, and have not found anything in other commentators in which I could quite acquiesce’. Modern readers are in no better position than Zabarella. Thus, Mignucci concludes that ‘it is difficult to make sense of the confused argument that Aristotle advances’ in Posterior Analytics 1.26. Smith holds that the argument is ‘very unsatisfactory’ and that ‘we cannot actually make c. 26 into a coherent account of per impossibile proof’. Likewise, Detel regards Aristotle’s argument as ‘garbled’ and, ‘as a matter of fact, unsuccessful’.

The plan of this paper is as follows. I begin by considering the preliminary part of Aristotle’s argument in chapter 1.26 (87a2–17), in which he explains how direct negative demonstrations differ from those by reductio (Section 2). Next, I turn to the core part of Aristotle’s argument, at 87a17–20 (Section 3). This part of the argument appeals to a relation of priority in nature among scientific propositions. I argue that this priority relation is determined by the order of terms in acyclic chains of immediate universal affirmations (Sections 4 and 5). Given this way of explicating priority in nature among scientific propositions, Aristotle’s argument in Posterior Analytics 1.26 turns out to be both coherent and successful (Section 6). I conclude by discussing how Aristotle’s argument from priority in

17 Philoponus in Post. An. 291.10–12.
18 Zabarella 1608: 976a.
19 Mignucci 1975: 559.
21 Detel 1993: ii 450.
nature fits with a remark he makes at 87a20–25 concerning the mereological structure exhibited by direct demonstrations (Section 7).

2. Direct negative demonstration vs. demonstration by reductio

In the *Analytics*, Aristotle takes demonstrations to be deductions in the three syllogistic figures. He focuses on deductions which consist of four kinds of categorical proposition:

- **AaB**  A belongs to all B
- **AeB**  A belongs to no B
- **AiB**  A belongs to some B
- **AoB**  A does not belong to some B

A demonstration is positive if its conclusion is an a- or i-proposition. A demonstration is negative (or, privative) if its conclusion is an e- or o-proposition.

In *Posterior Analytics* 1.25, Aristotle argues that positive demonstrations are better than negative ones. In chapter 1.26, he provides an argument to the effect that direct negative demonstrations are better than those by *reductio*. The argument is preceded by a preliminary discussion, in which Aristotle seeks to explain the difference between these two kinds of demonstration:

δεὶ δ’ εἰδέναι τίς ἡ διαφορὰ αὐτῶν. ἔστω δὴ τὸ Α μηδενὶ ὑπάρχον τῷ Β, τῷ δὲ Γ τὸ Β παντὶ ἀνάγκη δὴ τῷ Γ μηδενὶ ὑπάρχειν τὸ Α. οὕτω μὲν οὖν οὐν ιηθέντων δεικτικὴ ἢ στερητικὴ ἂν εἴη ἀπόδειξις ὅτι τὸ Α τῷ Γ οὐχ ὑπάρχει. ἡ δ’ εἰς τὸ ἀδύνατον ὧδ’ ἔχει. εἰ
We must understand what the difference between them is [i.e., between direct negative demonstration and demonstration by *reductio ad impossibile*]. Let A belong to no B and B to all C; it follows necessarily that A belongs to no C. If these premises are assumed, the privative demonstration that A does not belong to C will be ostensive. Demonstration leading to the impossible, on the other hand, proceeds as follows. If it is required to prove that A does not belong to B, we must assume that it does belong, and that B belongs to C; hence it follows that A belongs to C. But let it be known and agreed that this is impossible. Hence, A cannot belong to B. If, then, it is agreed that B belongs to C, it is impossible for A to belong to B.

In the first half of this passage, Aristotle considers a negative demonstration which is direct (or, ‘ostensive’).\(^{22}\) This demonstration takes the form of the syllogistic mood Celarent in the first figure:

\[ \text{AeB, BaC, therefore AeC} \]

\(^{22}\) In the *Prior Analytics*, Aristotle uses the term ‘ostensive’ (δεικτικός) to designate direct deductions as opposed to deductions by *reductio ad impossibile* and other ‘deductions from a hypothesis’ (συλλογισμοί ἐξ ὑποθέσεως); *Pr. An.* 1.7 29a31–3, 1.23 40b25–9, 41a21–37, 1.29 45a23–b15, 2.14 passim. In the *Posterior Analytics*, on the other hand, δεικτικός is sometimes used to designate positive as opposed to negative demonstrations (1.23 85a2, 1.25 86a32, 86b30–9). At 1.26, 87a5, the term is used in the former way to designate direct deductions (cf. Barnes 1993: 183).
In the second half of the passage, Aristotle goes on to consider a demonstration by *reductio*. One might have expected him to discuss a demonstration by *reductio* that derives the same conclusion as the preceding direct demonstration, AeC.23 He would then be able to compare the two demonstrations with respect to their explanatory value regarding this conclusion. As reasonable as this strategy may seem, it is not the one adopted by Aristotle in *Posterior Analytics* 1.26. Instead, he opts to consider a demonstration by *reductio* that derives the conclusion ‘A does not belong to B’. Accordingly, the assumption for *reductio* is the proposition ‘A belongs to B’. Since Aristotle’s formulation of these propositions does not contain a quantifying expression such as ‘no’ or ‘all’, there is some ambiguity as to their quantity. The conclusion of the demonstration by *reductio* might be either an e-proposition or an o-proposition, while the assumption for *reductio* might be either an a-proposition or an i-proposition. Now, Aristotle takes the assumption for *reductio* to be the major premise of a deduction in the first figure, with the minor premise being BaC. Since there are no (valid) first-figure deductions with a major i-premise, it is clear that the assumption for *reductio* is not an i- but an a-proposition.24 Thus, the subordinate deduction initiated by the assumption for *reductio* is an instance of the first-figure mood Barbara:

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23 According to Philoponus, some ancient commentators criticized Aristotle for not discussing a demonstration by *reductio* that derives the same conclusion as the direct demonstration (Philop. *in Post. An.* 294.12–14). A similar criticism is expressed by Detel 1993: ii 456–7.

24 Pacius 1605: 488.
AaB, BaC, therefore AaC

Aristotle takes it that, in the demonstration by \textit{reductio}, the proposition AaC is known and agreed to be ‘impossible’. This allows him to conclude the \textit{reductio} by inferring the conclusion of the demonstration. The conclusion thereby obtained is the contradictory opposite of the assumption for \textit{reductio}. Given that this assumption is AaB, the conclusion inferred in the demonstration by \textit{reductio} is AoB.

Against this, a number of commentators have maintained that the conclusion of the demonstration by \textit{reductio} is not AoB but the universal negative proposition AeB. In adopting this view, they are presumably guided by the thought that the demonstration by \textit{reductio} is intended to derive the major premise of the direct demonstration. At the same time, they acknowledge that the assumption for \textit{reductio} is the a-proposition AaB. Thus Aristotle would seem to commit the fallacy of inferring not the contradictory but the contrary opposite of the assumption for \textit{reductio}. Yet it seems unlikely that Aristotle committed such a fallacy. In \textit{Prior Analytics} 2.11, he emphasizes that the conclusion of a deduction by \textit{reductio}

must be the contradictory opposite, not the contrary opposite, of the assumption for reductio. He regards this as a constraint on reductio that is generally accepted (ἐνδοξὸν). Some commentators try to alleviate this problem by arguing that, in the Posterior Analytics, Aristotle adopts a framework in which a- and e-propositions are treated as exhaustive alternatives, so that the falsehood of an a-proposition entails the truth of the corresponding e-proposition. This proposal, however, is problematic. Not only does Aristotle not give any indication of adopting such a framework in the Posterior Analytics. Doing so would also commit him to the implausible view that the e-proposition No animal is human is true, given that the a-proposition Every animal is human is false. Zabarella suggests that, in Posterior Analytics 1.26, Aristotle treats a- and e-propositions as exhaustive alternatives because he ignores particular propositions and focuses exclusively on universal propositions. There is, however, no evidence that Aristotle adopted such a restriction to universal propositions in Posterior Analytics 1.26. On the contrary, he considers third-figure deductions that involve particular propositions in Posterior Analytics 1.14, and he discusses demonstrations of the form Bocardo that involve o-propositions in chapters 1.21 and 1.23.

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26 Pr. An. 2.11 62a11–19; see also 2.12 62a28–32.
29 Zabarella 1608: 973a–b; similarly, Barnes 1993: 188.
30 Post. An. 1.14 79a27–8, 1.21 82b21–8, 1.23 85a7–12. Barnes (1993: 173 and 183) excises the references to Bocardo at 82b21–8 and 85a7–12. However, his arguments for these excisions are not convincing (see Crager 2015: 104–20, Malink 2017: 184–5 nn. 63–4).
It is, therefore, preferable to take the conclusion of the demonstration by *reductio* to be the o-proposition $AoB$ rather than the e-proposition $AeB$. After all, Aristotle never asserts that the demonstration by *reductio* is intended to derive the universal negative premise of the direct demonstration. It may seem natural to suppose that the demonstration by *reductio* is meant to derive this premise. But, as we have seen, we cannot presume that Aristotle’s argument in chapter 1.26 conforms to our expectations.

Having specified the two demonstrations, Aristotle points out a difference that obtains between them concerning the epistemic status of their negative premise:

Thus the terms are similarly arranged in both demonstrations, but there is a difference as to which of the two privative propositions is more known, the one that A does not belong to B or the one that A does not belong to C. When the conclusion that it is not the case is more known, the demonstration leading to the impossible comes about; and when the proposition in the deduction is more known, the demonstrative proof comes about.

In the first sentence of this passage, Aristotle notes that the two demonstrations exhibit a similar arrangement of terms. Thus, he takes the two demonstrations to be related in that they are based on the same underlying structure of terms. In particular, both demonstrations
assume the truth of BaC and of negative propositions concerning AB and AC.\textsuperscript{31} There is, however, a difference as to ‘which of the two’ (ὅποτερα) negative propositions is more known. One of the two negative propositions is identified by Aristotle as ‘the conclusion that it is not the case’\textsuperscript{32}. This is the conclusion of the direct demonstration, AeC. The other

\begin{footnotesize}
\bibitem{Ross1949}

\bibitem{Tredennick1960}
tὸ συμπέρασμα … ὅτι οὐκ ἔστιν, 87a15. In this phrase, τὸ συμπέρασμα refers to the conclusion of the direct demonstration, AeC, while the qualification ὅτι οὐκ ἔστιν indicates that this conclusion is a negative proposition (Philoponus \textit{in Post. An.} 297.1–13, Tredennick 1960: 151, Mignucci 1975: 563, Barnes 1993: 41, Detel 1993: i 53, Tricot 2012: 146, Pellegrin 2005: 209 and 388 n. 6). On the other hand, some commentators adopt an alternative reading on which τὸ συμπέρασμα at 87a15 refers to the proposition AaC which appears as the conclusion of the subordinate deduction initiated by the assumption \textit{for reductio}. On this reading, the clause ὅταν μὲν οὖν ᾖ τὸ συμπέρασμα γνωριμώτερον ὅτι οὐκ ἔστιν means: ‘when it is more known that the conclusion AaC is not (i.e., is false)’; Zabarella 1608: 978d, Pacius 1605: 489, Maier 1900a: 232–3 n. 1, Mure 1928: ad loc., Crivelli 2011: 165. The former reading is, in my view, preferable because it fits the context better than the latter reading. For one thing, since συλλογισμός at 87a16 refers to the direct demonstration, it is natural to take συμπέρασμα at 87a15 to refer to the conclusion of this demonstration rather than to the conclusion of another deduction. Moreover, the three occurrences of συμπέρασμα at 87a18–20 refer to the conclusion of the direct demonstration, AeC. Finally, the two readings differ in the interpretation of the phrase οὐκ ἔστιν at 87a15. On the former reading, this phrase serves to demarcate negative from affirmative categorical propositions. On the latter reading, the phrase is used to indicate that a categorical proposition does not hold (or, is false), whether this proposition is affirmative or negative (see, e.g., \textit{Pr. An.} 2.2 53b12–13, 2.4 57b1–2). The former use of οὐκ ἔστιν appears at 1.26 87a26 (and 1.2 72a20, 1.23 84b30–1). By contrast, the latter use of οὐκ ἔστιν does not appear elsewhere in \textit{Posterior Analytics} 1.26.
\end{footnotesize}
negative proposition is identified by Aristotle as the one ‘in the deduction’. This is the premise of the direct demonstration, AeB.

Aristotle takes it that in some cases AeB is more known than AeC, while in other cases the latter proposition is more known than the former. Since the relation of being more known in nature is not variable in this way, he is presumably referring to the varying degree to which each of these two propositions is known to the demonstrator.33 Thus, given the truth of AeB, BaC, and AeC, Aristotle is describing the conditions under which a demonstrator will choose to employ either the direct demonstration or the one by reductio. If AeB is more known to the demonstrator than AeC, she will choose the direct demonstration establishing the latter proposition on the basis of the former. If, on the other hand, AeC is more known to the demonstrator than AeB, she will choose the demonstration by reductio. In this case, it is not possible for the demonstrator to derive the universal negative proposition AeB, but at least she is able to establish its particular counterpart, AoB.

In the demonstration by reductio, the proposition AaC is ‘known and agreed’ to be impossible (87a9–10). This is because its contrary opposite, AeC, is known and accepted by the demonstrator and her interlocutors.34 Thus, the demonstrator accepts both AeC and BaC, and uses the method of reductio to establish the conclusion AoB, as follows:

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Demonstration by reductio ad impossibile

1. AeC  (premise)
2. BaC  (premise)
3. AaB  (assumption for reductio)
4. BaC  (iterated from 2)
5. AaC  (from 3, 4, by Barbara)
6. AoB  (reductio: 1, 3–5)

In this derivation, lines 1 and 2 contain the two premises accepted by the demonstrator. Line 3 contains the assumption for reductio, and line 6 the conclusion of the demonstration, AoB. The inference from the two premises to the conclusion takes the form of the third-figure mood Felapton:

AeC, BaC, therefore AoB

A similar proof by reductio in which the premises and the conclusion constitute an instance of Felapton is given by Aristotle in Posterior Analytics 1.16. After positing the minor premise BaC (80a28–9), Aristotle reasons as follows:

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35 The step of reductio in line 6 relies on the fact that the conclusion of the subordinate deduction, AaC, is the contrary opposite of an accepted premise, AeC. Aristotle allows for the subordinate deduction in a reductio ad impossibile to be concluded, not only by the contradictory opposite of an accepted premise, but also by the contrary opposite of such a premise (e.g., Pr. An. 1.2 25a17–19, 1.7 29a36–9; see Thom 1981: 39–41, Crivelli 2011: 156–8).
That which belongs to no C will not belong to all B either; for if it belongs to B, it also belongs to C, but it was assumed that it does not belong.

Aristotle makes it clear that the conclusion of this proof by *reductio* is the o-proposition that A does not belong to all B (οὐδὲ τῷ Β παντὶ ὑπάρξει). If I am correct, he presents a proof of the same form, establishing an instance of Felapton, in his discussion of demonstration by *reductio* in chapter 1.26.

3. Aristotle’s argument from priority in nature

So far Aristotle has compared the two demonstrations with respect to what is more known to the demonstrator. He now goes on, in what is the core argument of the chapter, to compare them with respect to what is prior in nature:

By nature, however, the proposition that A does not belong to B is prior to the proposition that A does not belong to C. For the things from which a conclusion

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derives are prior to the conclusion; and that A does not belong to C is the conclusion, whereas that A does not belong to B is that from which the conclusion derives.

In this passage, Aristotle claims that the proposition AeB is prior in nature to AeC. He takes this claim to help establish the main thesis of the chapter, that direct negative demonstrations are better than demonstrations by *reductio* because the former, but not the latter, proceed from premises which are prior in nature to the conclusion (87a25–8). In the direct negative demonstration discussed by Aristotle, the premise AeB is prior in nature to the conclusion, AeC. In the demonstration by *reductio*, on the other hand, the premise AeC is not prior in nature to the conclusion, AoB. Instead, Aristotle seems to regard AeC as posterior in nature to AoB. At least, this is suggested by his remark that demonstrations by *reductio* proceed from premises which are posterior in nature (87a27). Thus, there is a clear sense in which the direct negative demonstration considered in 1.26 is better than the demonstration by *reductio*.

This argument from priority in nature gives rise to a number of questions. One of them, pointed out by Zabarella, concerns Aristotle’s discussion of *reductio ad impossibile* in *Prior Analytics* 2.14.\(^{37}\) In this chapter, Aristotle argues that every conclusion that is deducible from given premises by means of *reductio ad impossibile* can also be deduced from these premises without *reductio* by means of a direct deduction.\(^{38}\) Thus, he maintains that ‘whatever is proved through the impossible can also be concluded ostensively’ (62b40). For example, consider the following derivation by *reductio*:

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\(^{37}\) Zabarella 1608: 975f–976a.

\(^{38}\) *Pr. An.* 2.14 62b38–63b13; see also 1.29 45a23–b8.
1. CaA (premise)
2. CoB (premise)
3. AaB (assumption for reductio)
4. CaA (iterated from 1)
5. CaB (from 3, 4, by Barbara)
6. AoB (reductio: 1, 3–5)

Aristotle points out that the conclusion in line 6 of this derivation can be deduced directly from the premises in lines 1 and 2 without the use of reductio, by applying the second-figure mood Baroco (2.14 63a14–16). Thus, the derivation can be turned into a direct deduction simply by omitting the steps in lines 3–5. In the same way, the conclusion of the demonstration by reductio discussed in Posterior Analytics 1.26 can be deduced from the two premises directly without reductio, by applying the third-figure mood Felapton. In the resulting direct negative demonstration, the premise AeC fails to be prior in nature to the conclusion, AoB. Hence, there is a direct negative demonstration in which the premises are not prior in nature to the conclusion.

Conversely, Aristotle asserts in Prior Analytics 2.14 that, whenever a conclusion is deducible from given premises by means of a direct deduction, it can also be deduced from them by reductio ad impossibile (63b14–18). Thus, for example, the conclusion of Aristotle’s

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39 See Ross 1949: 455.
40 See also Pr. An. 1.29 45a26–7.
direct demonstration in Celarent can be deduced from the same premises by reductio. The result is a demonstration by reductio in which the premises, AeB and BaC, are prior in nature to the conclusion, AeC. Thus, given the interchangeability of direct deduction and reductio ad impossibile stated by Aristotle in Prior Analytics 2.14, it is not clear how he can maintain that direct negative demonstrations differ from those by reductio in that they proceed from premises which are prior in nature to the conclusion.

In view of this problem, Gisela Striker has suggested that Aristotle’s argument in Posterior Analytics 1.26 does not turn on the priority relations that obtain, or fail to obtain, between the premises and the conclusion in the two kinds of demonstration. Instead, she suggests, Aristotle’s argument turns on the way in which the conclusion is inferred from the premises in each case:

As Aristotle himself shows in An. Pr. B 14, one can form a ‘genuine’ premise pair for the demonstrandum from the true premise of the syllogism [i.e., the true premise of the direct deduction initiated by the assumption for reductio] and the negation of the impossible conclusion. However, Aristotle does not regard this further syllogism as a part of the reductio, but as a different proof of the same demonstrandum – in the reductio the proposition to be proved is inferred ‘from a hypothesis’ … Aristotle’s argument for the superiority of ‘categorical’ proofs presumably relies on the fact that reductio proofs are not purely syllogistic. (Striker 1977: 318)

Striker is drawing attention to the fact that Aristotle regards arguments by *reductio ad impossibile* as ‘deductions from a hypothesis’.\(^{42}\) According to Aristotle, every argument by *reductio* contains a part that can be analyzed as a direct deduction in the three syllogistic figures. This part of the argument appears within the subordinate derivation initiated by the assumption for *reductio*.\(^{43}\) In Aristotle’s view, no such syllogistic analysis is available for the final step of the argument, in which the assumption for *reductio* is discharged and the desired conclusion inferred. Aristotle describes this latter inference as ‘from a hypothesis’ and acknowledges that it cannot be justified within his theory of syllogistic moods in the three figures.\(^{44}\) According to Striker, this is what underlies Aristotle’s argument in *Posterior Analytics* 1.26.\(^{45}\)

Aristotle may well have accepted the considerations put forward by Striker, taking them to show that direct demonstrations are superior to those by *reductio*. Nonetheless, there is little evidence to suggest that Aristotle in fact appeals to these considerations in chapter 1.26. He does not, in this chapter, discuss the way in which the conclusion is derived from the premises in a demonstration by *reductio*. Nor does he allude to the fact that this derivation is ‘from a hypothesis’. Instead, he focuses on the relations of priority in nature that obtain

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\(^{42}\) *Pr. An.* 1.23 40b23–9.


\(^{44}\) *Pr. An.* 1.23 41a34, 1.44 50a29–32.

\(^{45}\) Similar suggestions have been made by Themistius in *Post. An.* 37.3–7 and Crivelli 2011: 170 n. 157.
between the premises and the conclusion of the respective demonstrations. This is difficult to explain on the interpretation proposed by Striker. For these relations of priority obtain, or fail to obtain, between the premises and the conclusion regardless of how the latter is derived from the former. Thus, as Barnes has pointed out, ‘Aristotle’s argument does not turn on the nature of reductio as such’. Moreover, Striker’s interpretation does not sit well with the elaborate exposition of the two kinds of demonstration given by Aristotle in the first half of chapter 1.26 (87a2–17). For, if Aristotle had in mind the point attributed to him by Striker, he could have provided much more straightforward examples of the two kinds of demonstration than he does. For instance, he could have chosen two examples in which the same conclusion is derived from the same premises. In fact, he could have omitted the examples altogether, since the point identified Striker can easily be stated in a general manner without appealing to any particular examples.

Thus, we are left with Zabarella’s problem of how to reconcile Aristotle’s argument in 1.26 with his treatment of reductio in Prior Analytics 2.14. To address this problem, it is helpful to take a closer look at the syllogistic framework employed in each chapter. In Prior Analytics 2.14, Aristotle takes for granted the fourteen syllogistic moods in the three figures that he established in Prior Analytics 1.1–6:

First figure: Barbara, Celarent, Darii, Ferio
Second figure: Cesare, Camestres, Festino, Baroco
Third figure: Darapti, Disamis, Datisi, Felapton, Ferison, Bocardo

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46 Barnes 1993: 188.
In *Prior Analytics* 2.14, Aristotle takes each of these moods to license a direct deduction from
the two premises to the conclusion. With respect to this system, he argues in 2.14 that any
conclusion that is deducible from given premises by *reductio ad impossibile* can also be
deducted from these premises by means of a direct deduction using the fourteen moods.\(^{47}\)
Thus, the rule of *reductio ad impossibile* is redundant in Aristotle’s full system of fourteen
moods.

Crucially, this does not mean that the rule of *reductio* is redundant in Aristotle’s
theory of the assertoric syllogism as a whole. In *Prior Analytics* 1.1–7, Aristotle presents two
deductive systems in which the rule of *reductio* plays an indispensable role. The first of these
systems, expounded in *Prior Analytics* 1.2 and 1.4–6, includes among its principles the four
first-figure moods which Aristotle regards as ‘perfect’ (chapter 1.4). Moreover, the system
includes three conversion rules (chapter 1.2), and a rule of *reductio ad impossibile*:\(^{48}\)

1. Perfect moods:  
   - AaB, BaC, therefore AaC \hspace{1cm} (Barbara)
   - AeB, BaC, therefore AeC \hspace{1cm} (Celarent)
   - AaB, BiC, therefore AiC \hspace{1cm} (Darii)
   - AeB, BiC, therefore AoC \hspace{1cm} (Ferio)

2. Conversion rules:  
   - AeB, therefore BeA \hspace{1cm} (e-conversion)
   - AiB, therefore BiA \hspace{1cm} (i-conversion)
   - AaB, therefore BiA \hspace{1cm} (a-conversion)

3. Rule of *reductio ad impossibile*

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In *Prior Analytics* 1.5–6, Aristotle employs this system to establish the validity of syllogistic moods in the second and third figures. In most cases, he does so by a direct deduction, employing the perfect first-figure moods and conversion rules. There are, however, two valid moods that cannot be established in this way by means of a direct deduction: Baroco and Bocardo. Aristotle establishes these two moods by *reductio ad impossibile*, using the perfect mood Barbara. Thus, the rule of *reductio* is not redundant in this deductive system, since Baroco and Bocardo cannot be established directly but only by *reductio*.50

In *Prior Analytics* 1.7, Aristotle goes on to present a second deductive system, in which the rule of *reductio* plays an even more prominent role. In this system, Aristotle no longer includes the particular first-figure moods Darii and Ferio in the list of principles. Instead, he establishes these moods by *reductio ad impossibile*, using the first-figure mood Celarent.51 In addition, Aristotle uses *reductio ad impossibile* to derive the rules of *i*- and *a*-conversion from

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49 Pr. An. 1.5 27a36–27b1, 1.6 28b15–20.

50 Neither Baroco nor Bocardo can be established by means of a direct deduction in Aristotle’s deductive system because it does not include a conversion rule for o-propositions (see Alexander in Pr. An. 83.12–25, Łukasiewicz 1957: 54). In the case of Bocardo, Aristotle mentions an alternative proof that employs the method of ecthesis instead of *reductio* (Pr. An. 1.6 28b20–1). However, he does not mention a proof by ecthesis for Baroco, and it is not clear whether such a proof is available. For example, the formulation of the rule of ecthesis given by Parsons (2014: 36–7) allows for a proof of Bocardo but not Baroco (similarly, Ebert & Nortmann 2007: 333–7). Without appealing to ecthesis, the only way for Aristotle to establish the validity of Baroco and Bocardo in his system is by means of *reductio*.

that of e-conversion.\textsuperscript{52} Thus, the deductive system presented by Aristotle in \textit{Prior Analytics} 1.7 can be taken to rest on the following principles:

1. Perfect moods: $AaB$, $BaC$, therefore $AaC$  
   \textbf{(Barbara)}
   $AeB$, $BaC$, therefore $AeC$  
   \textbf{(Celarent)}

2. Conversion rule: $AeB$, therefore $BeA$  
   \textbf{(e-conversion)}

3. Rule of \textit{reductio ad impossibile}

Just like the previous deductive system, this streamlined system is strong enough to derive all fourteen moods of Aristotle’s assertoric syllogistic.\textsuperscript{53} There are, however, only four moods that can be established by means of a direct deduction in this system, namely, the four universal moods of the assertoric syllogistic: Barbara, Celarent, Cesare, and Camestres. The remaining ten moods, including Felapton, cannot be established directly but only by \textit{reductio}. Thus, in this system, a significant portion of the assertoric syllogistic depends on \textit{reductio ad impossibile}. At the same time, the system is significant because it allows Aristotle to reduce all syllogistic moods to Barbara and Celarent. Thus, when Aristotle summarizes the results of

\textsuperscript{52} \textit{Pr. An}. 1.2 25a17–22. Aristotle’s proof by \textit{reductio} of a-conversion relies on the assumption that a-propositions are contrary opposites of the corresponding e-propositions (see n. 35 above; cf. Malink 2013: 40–1).

\textsuperscript{53} This presupposes that the rule of \textit{reductio ad impossibile} allows for nested applications of \textit{reductio} within another \textit{reductio} (as in the system given by Smiley 1973: 141–2). Aristotle employs such nested applications of \textit{reductio}, e.g., at \textit{Pr. An}. 1.15 34b2–6 and \textit{Metaph. Θ 4} 1047b14–26 (see Fine 2011: 1023–8, Rosen & Malink 2012: 234–42, Malink & Rosen 2013: 971–3).
Prior Analytics 1.1–7 in chapter 1.23, he emphasizes that in the streamlined system ‘every deduction is completed through the first figure and is reduced to the universal deductions in this figure’ (1.23 41b3–5).54

All told, then, the three syllogistic frameworks employed by Aristotle in the Prior Analytics differ from one another in the extent to which they rely on reductio ad impossibile. In the full system of the assertoric syllogistic, reductio is redundant and is not needed to establish any moods. In the deductive system of chapters 1.4–6, reductio is not redundant but is needed to establish Baroco and Bocardo. In the system of chapter 1.7, reductio is needed to establish most syllogistic moods, with the exception only of the four universal moods.

In the Posterior Analytics, Aristotle does not specify the syllogistic framework that underlies his theory of demonstration. It is clear that, in chapters 1.14–26, he relies on some version of the syllogistic theory developed in the Prior Analytics, but he does not describe it in any detail. For the most part it is not necessary for Aristotle to do so in the Posterior Analytics. His account of demonstration is to a large extent compatible with different syllogistic frameworks, and entering into a discussion of them would be extraneous to his main aims in the Posterior Analytics. In chapter 1.26, however, it is important for Aristotle to demarcate the moods that rely on reductio ad impossibile from those that do not. This demarcation depends on exactly which syllogistic framework is adopted.

As we have seen, the full system of the assertoric syllogistic, in which all fourteen moods are taken to license direct deductions, does not sit well with Aristotle’s argument in *Posterior Analytics* 1.26. The same is true for the deductive system presented in *Prior Analytics* 1.4–6. For, in this system, the mood Felapton does not rely on reductio, but can be established by means of a direct deduction.\(^\text{55}\) Hence, in this system, Aristotle’s demonstration by reductio in *Posterior Analytics* 1.26 can be replaced by a direct deduction deriving the same conclusion from the same premises. By contrast, this problem does not arise in the streamlined system from *Prior Analytics* 1.7. For, in this system, Felapton cannot be established by means of a direct deduction but only by reductio. Hence, among the three systems, the one from chapter 1.7 fits best with Aristotle’s argument in *Posterior Analytics* 1.26. More generally, the argument fits with any variant of this system which yields the same demarcation between moods that rely on reductio and those that do not. For example, the argument might be taken to be based on a variant of this system in which Cesare and Camestres are not reduced by e-conversion to Celarent, but are posited as additional principles along with the first-figure moods Barbara and Celarent.\(^\text{56}\) In any case, all that is important for our purposes is that, in *Posterior Analytics* 1.26, Aristotle employs a deductive

\(^\text{55}\) *Pr. An.* 1.6 28a26–9.

\(^\text{56}\) This would be in accordance with the fact that, in the first book of the *Posterior Analytics*, Aristotle does not mention any conversion rules and does not undertake to reduce second- and third-figure moods to those in the first figure (Smith 1982a: 115–17 and 121–2, 1982b: 331–3).
system in which the only moods that can be established by a direct deduction are the purely universal moods, with all other moods relying on reductio.\footnote{Pace Lear (1980: 53), who maintains that in Posterior Analytics 1.26 every demonstration by reductio can be replaced by a direct demonstration deriving the same conclusion from the same premises. Lear suggests that, ‘when Aristotle comes to criticize proof per impossibile, in Posterior Analytics A26, all he can say is that the premisses which are prior in nature – those from which the conclusion can be proved directly – are not sufficiently familiar to us’ (Lear 1980: 53). This, however, is not correct. Aristotle’s point at 1.26 87a17–20 and 87a25–30 is not that the premises of demonstrations by reductio fail to be familiar to us, but that they fail to be prior in nature to the conclusion.}

Given such a deductive system, Aristotle’s thesis in 1.26 amounts to the claim that all demonstrations of the form Barbara, Celarent, Cesare, and Camestres proceed from premises that are prior in nature to the conclusion, whereas this is not the case for demonstrations that take the form of other syllogistic moods. In order to verify this claim, we need an account of what it is for one proposition to be prior in nature to another. In what follows, I provide such an account for both affirmative and negative propositions.

4. Priority in nature for a-propositions

As we have seen, when Aristotle speaks of ‘demonstration by reductio ad impossibile’ in Posterior Analytics 1.26, he uses the term ‘demonstration’ in a broader sense than the one introduced in his definition of demonstration in chapter 1.2. Otherwise, his claim that demonstrations by reductio do not proceed from premises that are prior in nature to the
conclusion would be contradictory. Accordingly, his claim that direct demonstrations do proceed from such premises would be trivially true, simply in virtue of the definition of what a demonstration is. What, then, is the intended reference of the term ‘demonstration’ in these claims?

Minimally, a demonstration is a deduction (συλλογισμός). Yet it is unlikely that Aristotle uses ‘demonstration’ in *Posterior Analytics* 1.26 in a sense that is so broad as to include all deductions. For Aristotle is aware that not every direct deduction proceeds from premises that are prior in nature to the conclusion. For example, as he points out in *Posterior Analytics* 1.6, there are deductions in which a true conclusion is deduced from false premises.\(^5\) Clearly these premises are not prior in nature to the conclusion. So Aristotle’s claim that all direct demonstrations proceed from premises that are prior in nature to the conclusion would be false if ‘demonstration’ is used to designate any deduction. Perhaps Aristotle uses ‘demonstration’ to designate all deductions in which the premises are true? Again, such a use of the term would seem too broad. For, in Aristotle’s view, demonstration is essentially tied to scientific knowledge (ἐπιστήμη), and there are many true propositions that fall outside the purview of scientific knowledge.\(^6\) Instead, Aristotle may be taken to use ‘demonstration’ in 1.26 to designate all deductions in which the premises are scientific propositions, that is, either indemonstrable premises or demonstrable theorems of a given

\(^{5\text{Post. An. 1.6 75a3–4; cf. Pr. An. 2.2–4.}}\)

\(^{6\text{For example, true propositions about chance events fall outside the purview of scientific knowledge and demonstration (Post. An. 1.30 87b19–27, Pr. An. 1.13 32b18–22).}}\)
science. On this use of the term, any deduction counts as a ‘demonstration’ as long as the premises are scientific propositions, whether or not the deduction satisfies the definition of demonstration given in chapter 1.2. While Aristotle does not explicitly introduce this broader sense of ‘demonstration’, it is a natural way for him to use the term. On the one hand, since the premises of a ‘demonstration’ are required to be scientific propositions, this use of the term guarantees that ‘demonstrations’ are restricted to those propositions that fall under the purview of a given science. On the other hand, this use of the term is broad enough to allow for a coherent rendering of Aristotle’s thesis in *Posterior Analytics* 1.26.

On this account, Aristotle’s thesis in 1.26 amounts to the following claim: in every deduction of the form Barbara, Celarent, Cesare, and Camestres in which the premises are scientific propositions, these premises are prior in nature to the conclusion; in deductions that are not of one of these forms, the premises need not be prior in nature to the conclusion even if they are scientific propositions. This is a substantive thesis, but it is not incoherent. As we shall see, the thesis can be verified provided a suitable characterization of priority in nature.

In *Posterior Analytics* 1.2–3, Aristotle states that the premises of genuine demonstrations are prior in nature to the conclusion. He does not, in these chapters, explain what it is for one proposition to be prior in nature to another, or how to determine when this relation obtains between two propositions. However, he provides some guidance on these

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60 *Post. An.* 1.2 71b19–72a5, 1.3 72b25–32; cf. n. 6 above.
questions in *Posterior Analytics* 1.19–25. In these chapters, Aristotle considers chains of terms connected by universal affirmations. In chapter 1.19, he describes these chains as follows:

> ἔστω δὴ τὸ Γ τοιοῦτον ὃ αὐτὸ μὲν μηκέτι ὑπάρχει ἄλλῳ, τούτῳ δὲ τὸ B πρῶτῳ, καὶ οὐκ ἔστιν ἄλλο μεταξύ. καὶ πάλιν τὸ E τῷ Z ὡσαύτως, καὶ τοῦτο τῷ B. (*Post. An.* 1.19 81b30–2)

Let C be such that it itself does not further belong to any other thing, and that B belongs to it directly, with nothing else between them. Again, let E belong to F in the same way, and F to B.

If universal affirmation is indicated by arrows pointing from the predicate to the subject term, the chain of universal affirmations described in this passage can be represented as follows:61

![Diagram](image)

In this diagram, each term belongs to its successor 'directly' (πρῶτῳ), that is, in such a way that there is no other term between them. Thus, each of these universal affirmations is immediate (ἄμεσος).62 For Aristotle, a scientific proposition is immediate just in case there is

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61 It is clear from the context that the unquantified occurrences of ὑπάρχειν in this passage are meant to indicate a-propositions (see 1.19 81b10–18).

62 See, e.g., 1.21 82b7, 1.22 84a35, 1.23 84b14, 84b22, 84b36–85a1, 1.25 86b31; cf. Philoponus in *Post. An.* 220.19–20, Owen 1889: 287, Mure 1928: ad 81b30.
no proposition that is prior in nature to it.63 As such, immediate propositions are
indemonstrable principles of a science.64 By contrast, propositions that are not immediate are
demonstrable from premises that are prior in nature to them. For example, in the following
passage, the a-proposition AaE is not immediate since it is demonstrable from AaD and DaE:

\[\text{(Post. An. 1.25 86a39–86b5)}\]

Let one demonstration show that A belongs to E through the middle terms B, C, and D, and let the other show that A belongs to E through F and G. Thus the proposition that A belongs to D and the one that A belongs to E are on a par. But that A belongs to D is prior to and more known than that A belongs to E; for the latter is demonstrated through the former.

In this passage, AaE is demonstrable through two distinct chains of middle terms:

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63 Post. An. 1.2 72a7–8.

64 Post. An. 1.2 72a7–8, 1.3 72b18–22.
The major term A is connected to the minor term E by two a-paths, that is, paths of immediate universal affirmations. In this setting, the propositions AaD and AaE are ‘on a par’ in the sense that both of them are demonstrable through the same number of middle terms, the former through B and C, and the latter through F and G. At the same time, Aristotle takes the proposition AaD to be prior in nature, and more known in nature, than AaE. Presumably, AaD is prior in nature to AaE because it is part of the a-path from A to E and is required for the demonstration of AaE, but not vice versa. For the same reason, the propositions AaC and CaE are prior in nature to AaE. For, just like AaD, these two propositions are part of the a-path from A to E, and are required for the demonstration of AaE through the middle terms B, C, and D.

In Aristotle’s syllogistic theory, the only way to deduce a universal affirmative conclusion AaB is by means of a-premises that constitute a path of universal affirmations

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66 It seems clear that πρότερον καὶ γνωριμώτερον at 1.26 86b3–4 means ‘prior in nature and more known in nature’; see Burnyeat 1981: 127–8.

67 This does not necessarily mean that the propositions AaC and CaE appear in the demonstration of AaE. In Posterior Analytics 2.18, Aristotle suggests that, when AaE is demonstrated through B, C, and D, the final step of the demonstration should employ the least universal middle term, D, inferring AaE from AaD and DaE (99b9–14). Likewise, AaD should be inferred from AaC and CaD. On this account, the propositions AaC and CaE do not appear in the demonstration of AaE. Nonetheless, the truth of these propositions is required for the demonstration of AaE through B, C, and D.
leading from A to B.\textsuperscript{68} If these a-premises are immediate, they constitute an a-path from A to B in the science under consideration. Any such a-path gives rise to a demonstration of AaB.\textsuperscript{69} All propositions that appear on such an a-path are prior in nature to AaE. On this account, an a-proposition CaD is prior in nature to an a-proposition AaB just in case there is an a-path from C to D which is a proper part of some a-path from A to B.

This characterization of priority in nature provides a natural way of spelling out the notion of priority that underlies Aristotle’s discussion in the passage just quoted. Of course, the characterization does not provide us with a criterion for deciding whether or not a given scientific a-proposition is prior in nature to another such proposition. For example, it does not allow us to decide whether the a-proposition \textit{All planets are non-twinkling} is prior in nature to the a-proposition \textit{All planets are near} or vice versa. This is because, in appealing to a-paths of immediate universal affirmations, the characterization presupposes a demarcation of the immediate a-propositions from those a-propositions that are not immediate in a given science. Given such a demarcation, the above characterization determines the relation of priority in nature for all a-propositions that fall under the purview of the science.


\textsuperscript{69} With respect to a-paths of immediate universal affirmations, Aristotle holds that, ‘when A belongs to B, then, if there is some middle term, it is possible to prove [i.e., demonstrate] that A belongs to B’ (1.23 84b19–20). Similarly, he writes: ‘terms so related to their subjects that there are other terms prior to them predicated of those subjects are demonstrable’ (ἀν πρότερα ἄττα κατηγορεῖται, ἔστι τούτων ἀπόδειξις, 1.22 83b33–4; transl. Mure 1928 modified). Thus, Aristotle maintains that, if there is an a-path of immediate universal affirmations from A to B, this a-path gives rise to a demonstration of AaB (see McKirahan 1992: 210).
Aristotle maintains that the relation of priority in nature is asymmetric: if one proposition is prior in nature to another, the latter is not prior in nature to the former. Therefore, in order for the above characterization to be adequate, it must conform to this requirement of asymmetry. Now, the characterization fails to be asymmetric if there are cycles of immediate α-propositions, such as:

\[ \text{AaB, BaC, CaA} \]

Given that each of these three propositions is immediate, AaB is prior in nature to AaC, since the shortest a-path from A to B is a proper part of the shortest a-path from A to C. At the same time, AaC is prior in nature to AaB, since the shortest a-path from A to C is a proper part of the following a-path from A to B:

Thus, the proposed characterization of priority in nature fails to be asymmetric if there are cyclic a-paths. There is, however, good reason to think that Aristotle intends to exclude cyclic a-paths in the first book of the *Posterior Analytics*. In particular, he can be viewed as

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70 Aristotle maintains that 'it is impossible for the same things at the same time to be both prior and posterior to the same things' *Post. An.* 1.3 72b27–8. Similarly, 2.15 98b16–21.
excluding such a-paths in a passage from chapter 1.22 in which he deals with what he calls ‘counterpredication’ (ἀντικατηγορεῖσθαι). The passage relies on the theory of predication developed in chapter 1.22, and reads as follows:

[i] ἔτι εἰ μή ἐστι τοῦτο τοῦτο ποιότης κἀκεῖνο τούτου, μηδὲ ποιότητος ποιότης, ἀδύνατον ἀντικατηγορεῖσθαι ἄλληλων οὕτως, ἀλλ’ ἀληθές μὲν ἐνδέχεται εἰπεῖν, ἀντικατηγορήσαι δ’ ἂληθώς οὐκ ἐνδέχεται. [ii] ἢ γάρ τοι ὡς ούσία κατηγορηθῆσαι, οἷον ἢ γένος ὡς ἢ διαφορά τοῦ κατηγορουμένου. . . . ὡς μὲν δὴ γένη ἄλληλων οὐκ ἀντικατηγορηθῆσαι· ἔσται γὰρ αὐτὸ ὑπὲρ αὐτὸ τι. [iii] οὐδὲ μὴν τοῦ ποιοῦ ἢ τῶν ἄλλων οὐδέν, ἂν μὴ κατὰ συμβεβηκὸς κατηγορηθῇ· πάντα γὰρ ταῦτα συμβέβηκε καί κατὰ τῶν οὐσιῶν κατηγορεῖται. (Post. An. 1.22 83a36–b12)

[i] If this is not a quality of that and that of this – a quality of a quality –, then it is impossible for one thing to be counterpredicated of another in this way. It is possible to make a true statement, but it is not possible to counterpredicate truly. [ii] For either it will be predicated as substance, i.e. being either the genus or the differentia of what is predicated. . . . Surely they will not be counterpredicated of one another as genera; for then something would be just what is some of itself. [iii] Nor will anything be counterpredicated \(^{71}\) of a quality or the other kinds of thing – unless it is predicated accidentally; for all these things are accidents, and they are predicated of substances.

\(^{71}\) It is natural to supply ἀντικατηγορηθῆσαι, from 83b9, as the main verb of this sentence; Waitz 1846: 357, Mure 1928: ad loc., Tredennick 1960: 123, Barnes 1975: 35, Seidl 1987: 107, Tricot 2012: 120. On the other hand, some authors take κατηγορηθῆσαι to be the main verb of the sentence; Mignucci 1975: 468–9, Barnes 1993: 32, Pellegrin 2005: 177 and 381 n. 14. Moreover, I take τοῦ ποιοῦ ἢ τῶν ἄλλων to be the genitive object of the verb of the sentence; Tredennick 1960: 123, Seidl 1987: 107, Barnes 1993: 32, Pellegrin 2005: 177.
This passage presents a number of difficulties. Nonetheless, it seems clear that its main focus is on counterpredication, that is, on cases in which a term $A$ is predicated of $B$ and $B$ is predicated of $A$.\footnote{Waitz 1846: 356–7, Mure 1928: ad loc., Ross 1949: 578–9, Hamlyn 1961: 119–20, Barnes 1975: 170, Lear 1979: 214, 1980: 31, Tricot 2012: 118–20; pace Mignucci 1975: 460–9, Barnes 1993: 177–8} Aristotle considers various putative cases of counterpredication and argues that they are not admissible in the theory of predication employed in chapter 1.22. In point [i] of the passage, he states that two terms cannot be counterpredicated of one another in such a way that one is a quality of the other and vice versa. For example, if *pale* and *musical* are accidents in the category of quality, they are not counterpredicated of one another. Aristotle admits that there is a sense in which it is true to say that the pale thing is musical and the musical thing is pale. He insists, however, that this is not an instance of true counterpredication. Thus, if both $A$ and $B$ are accidents in the category of quality, it is not the case that $A$ is predicated of $B$ and $B$ is predicated of $A$.

In point [iii] of the passage, Aristotle strengthens this claim, asserting that nothing is counterpredicated of an accident that is in the category of quality or in any of the other non-substance categories. Thus, if $A$ is such an accident and is predicated of $B$, then $B$ is not predicated of $A$. For example, if *pale* is predicated of *log*, then the latter is not predicated of the former. Again, Aristotle admits that there is a sense in which it is true to say that the pale hand, Barnes (1975: 35, 1984: i 136) takes it to be a partitive genitive depending on οὐδέν ('no case of quality or the other kinds of predication').
thing is a log (1.22 83a1–3). Such predications, however, are merely accidental.\textsuperscript{73} In *Posterior Analytics* 1.22, Aristotle makes it clear that accidental predications are not admissible in demonstrations (83a18–21). Thus accidental predications, in which something is predicated of an accident, are excluded from consideration in the theory of predication expounded in chapters 1.19–22. Consequently, nothing is counterpredicated of an accident.

In point [ii], Aristotle considers the question whether counterpredication can obtain between items which are not accidents but are predicated essentially ‘as substance’. He asserts that two items cannot be counterpredicated of one another in such a way that they are genera of one another. It is not immediately clear whether Aristotle also intends to exclude essential counterpredications between a definiendum and its definiens, such as *man* and *biped animal*. Philoponus argues that Aristotle intends to exclude such counterpredications on the grounds that in these cases the counterpredicating items are not distinct but the same.\textsuperscript{74} If this is correct, Aristotle can be taken to exclude any essential counterpredications.

In accordance with this, it is widely agreed that in the passage just quoted Aristotle intends to exclude the possibility of any counterpredication in which A is predicated of B and vice versa.\textsuperscript{75} Now, in *Posterior Analytics* 1.19–22, Aristotle takes the relation of predication to coincide with that of universal affirmation. Specifically, he seems to hold that A is predicated

\textsuperscript{73} Post. An. 1.22 83a1–18; cf. 1.19 81b23–9, Pr. An. 1.27 43a33–6.

\textsuperscript{74} Philoponus in Post. An. 246.14–24.

of B just in case AaB is a scientific proposition.\textsuperscript{76} Hence, by excluding counterpredication, Aristotle excludes a-paths of the form AaB, BaA. More generally, he excludes any cyclic a-paths of the form A,aA, a,Aa, \ldots, A,aA.\textsuperscript{77} For, the existence of such an a-path implies that both A,aA and a,Aa are scientific propositions, and hence that A and A are counterpredicated of one another.

In \textit{Posterior Analytics} 1.22, Aristotle asserts that all demonstrations in Barbara are based on chains of ‘direct’, or, immediate, universal affirmations. In a demonstration by Barbara, ‘it is necessary that there is some term of which something is predicated directly (πρῶτον), and another term of this; and this must come to a stop, and there must be terms which are no longer predicated of anything prior and of which nothing else prior is predicated’ (83b28–31). At the same time, Aristotle notes that such chains of immediate universal affirmations are not available in the presence of counterpredicated terms. For, ‘among counterpredicated terms, there is none of which any term is predicated directly (πρῶτου) or of which it is predicated last; for in this respect at least every such term is related

\textsuperscript{76} In chapters 1.19–22, Aristotle uses the verb ‘to belong’ (ὑπάρχειν) to express scientific a-propositions (e.g., 1.19 81b10–18, 81b30–7, 82a2–8, 1.21 82b19–21, 82b34–5, 1.22 83b25, 1.23 84b19–22). At the same time, he uses the verb ‘to be predicated’ (κατηγορεῖσθαι) interchangeably with ‘to belong’ (see, e.g., 1.19 81b34, 82a1, 1.20 82a24–9, 1.23 84b32). See also the transition from υπάρχειν at 1.21 82b34–5 to κατηγορεῖσθαι at 1.22 82b37–83a21 and κατηγορία at 84a1 and 84a39–b1. Accordingly, Aristotle maintains that there is an infinite regress of demonstrations in Barbara just in case there is an infinite chain of predications between two terms (1.19 82a2–8, 1.22 83b32–84a6, 84a29–b2; see Lear 1979: 201–8, 1980: 18–26).

to every other in the same way’ (1.19 82a15–17). Thus, in maintaining that all demonstrations in Barbara are based on chains of immediate universal affirmations, Aristotle in effect excludes counterpredication from the domain of demonstration. Consequently, the predicative structure underlying universal affirmative demonstrations in any given science does not contain any cyclic a-paths.

Thus, any science determines a structure of acyclic immediate universal affirmations. These a-structures can be defined as follows, where immediate universal affirmation is indicated by an arrow (→):78

**DEFINITION 1:** An *a-structure* consists of a set of terms equipped with a binary acyclic relation, →. Thus, there are no terms $A_1, A_2, ..., A_n$ ($n \geq 1$) such that $A_i \rightarrow A_{i+1}$ (1 ≤ $i$ ≤ $n$–1) and $A_n \rightarrow A_1$.

An a-path is a sequence of terms connected by immediate universal affirmations:

**DEFINITION 2:** Given an a-structure, an *a-path* is a sequence of terms $A_1, A_2, ..., A_n$ ($n > 1$) such that $A_i \rightarrow A_{i+1}$ for all 1 ≤ $i$ ≤ $n$–1.

78 Definition 1 does not provide an exhaustive characterization of the a-structures considered by Aristotle in the first book of the Posterior Analytics. In addition to acyclicity, these a-structures satisfy a number of other conditions. For example, they do not contain any infinite a-paths (Post. An. 1.22 84a7–11, 84a29–b2). Moreover, they do not contain any terms such that $A \rightarrow B$, $A \rightarrow C$, and $C \rightarrow B$ (see n. 69). Since, however, these additional conditions are not essential to the argument of this paper, I set them aside.
Since a-structures are acyclic, no term appears twice on an a-path. Thus, a-paths can be represented by acyclic directed graphs such as the following:

![Diagram of an a-path]

This diagrammatic representation of a-paths is in accordance with the spatial terminology employed by Aristotle in the first book of the *Posterior Analytics* to describe scientific a-propositions. For example, he refers to these propositions as ‘intervals’ (διαστήματα). In doing so, he compares a scientific a-proposition AaB to a one-dimensional region of space bounded by two points, with the terms A and B corresponding to the two endpoints. Accordingly, he refers to the intervals that represent immediate a-propositions as ‘indivisible’ (ἀδιαίρετον). If an interval AaB is indivisible in this sense, then A is said to belong to B atomically (ἀτόμως). Accordingly, an indemonstrable interval is called ‘atomic’ (ἄτομον). On the other hand, if an interval is demonstrable, it is called ‘divisible’ (διαιρετόν).

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79 *Post. An.* 1.21 82b7–8, 1.22 84a35; 1.23 84b14. Similarly, *Pr. An.* 1.4 26b21, 1.15 35a12 and 35a31; 1.18 38a4, 1.25, 42b9–10, 2.2, 53b20.

80 *Post. An.* 1.22 84a35, 1.23 84b35.

81 *Post. An.* 1.15 79a33–4, 1.16 80a3, 80a12, 1.17 80b17.


83 *Post. An.* 1.22 84a35.
above a-path, the proposition AaB is represented by a divisible interval in which the terms A and B are separated by eight atomic intervals.

In *Posterior Analytics* 1.23, Aristotle refers to the immediate a-premises that are represented by the atomic intervals between A and B as elements (στοιχεῖα) of the a-proposition AaB:

φανερὸν δὲ καὶ ὅτι, ὅταν τὸ Α τῷ Β ὑπάρχῃ, εἰ μὲν ἔστι τι μέσον, ἔστι δεῖξαι ὅτι τὸ Α τῷ Β ὑπάρχει, καὶ στοιχεῖα τούτου ἐστὶ ταῦτα …· αἱ γὰρ ἄμεσοι προτάσεις στοιχεῖα. (*Post. An.* 1.23 84b19–22)

It is evident that when A belongs to B, then if there is some middle term it is possible to prove that A belongs to B, and the elements of this [conclusion] are these [premises] …, for the immediate premises are elements.

For Aristotle, an element is an indivisible constituent of that of which it is an element. Thus, he views immediate a-premises as indivisible constituents of the a-propositions demonstrated from them. Since Aristotle takes elements to be prior to the things of which they are elements, immediate a-premises are prior to the universal affirmative theorems demonstrated from them. For the same reason, any a-proposition that corresponds to a non-atomic proper part of the interval between A and B will be prior to the theorem AaB.

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84 See Malink 2017: 174–86.
85 *Metaph.* Δ 3 1014a26–30.
86 ‘The element is prior to the things of which it is an element’, *Metaph.* A 4 1070b2–3.
More generally, according to the account proposed above, an a-proposition \(CaD\) is prior in nature to an a-proposition \(AaB\) just in case some a-path from \(C\) to \(D\) is a proper part of some a-path from \(A\) to \(B\). Given that a-structures are acyclic, this relation of priority in nature is both transitive and irreflexive.\(^{87}\) As such, it is a well-defined priority relation that does justice to Aristotle’s requirement of asymmetry. Moreover, this relation of priority in nature allows us to verify part of Aristotle’s claim in *Posterior Analytics* 1.26 that the premises of direct demonstrations are prior in nature to the conclusion. In particular, it allows us to verify the claim that, whenever the premises of a deduction in Barbara are scientific propositions, these premises are prior in nature to the conclusion. To see this, consider an instance of Barbara in which the two premises, \(AaB\) and \(BaC\), are scientific propositions. The premises are either immediate propositions or demonstrable theorems in a given science. Hence, there is an a-path from \(A\) to \(B\), and an a-path from \(B\) to \(C\). The concatenation of these a-paths is an a-path from \(A\) to \(C\). The first two a-paths are proper parts of the third a-path. Hence, each of the premises \(AaB\) and \(BaC\) is prior in nature to \(AaC\).

In order to validate Aristotle’s claim for all direct demonstrations, it remains to verify it for the moods Celarent, Cesare, and Camestres. To this end, we need an account of what it is for e-propositions to be prior in nature to one another.

\(^{87}\) Clearly, this relation is transitive: if \(AaB\) is prior in nature to \(CaD\), and the latter to \(EaF\), then \(AaB\) is prior in nature to \(EaF\). To see that the relation is irreflexive, suppose that \(AaB\) is prior in nature to itself. This means that some a-path from \(A\) to \(B\) is a proper part of some a-path from \(A\) to \(B\). Hence, either \(A\) or \(B\) occur twice on the latter a-path from \(A\) to \(B\), thus violating the condition of acyclicity.
5. Priority in nature for e-propositions

In addition to immediate a-propositions, Aristotle countenances immediate e-propositions in the *Posterior Analytics*. If an e-proposition is immediate, it is atomic in the sense that there is no middle term through which it can be demonstrated:

\[ \text{ωσπερ δὲ υπάρχειν τὸ Α τῷ Β ἐνεδέχετο ἀτόμως, οὔτω καὶ μὴ υπάρχειν ἐγχωρεῖ. λέγω δὲ τὸ ἀτόμως υπάρχειν ή μὴ υπάρχειν τὸ μή εἶναι αὐτῶν μέσον: οὔτω γάρ οὐκέτι ἔσται κατ’ ἄλλο τὸ υπάρχειν ή μὴ υπάρχειν. (Post. An. 1.15 79a33–6) } \]

Just as it is possible for A to belong to B atomically, so it is also possible for it atomically not to belong. By atomically belonging or not belonging I mean that there is no middle term for them; for, in this case, they no longer belong or do not belong by virtue of something else.

If AeB is an immediate e-proposition, then B is the ‘first’ (πρώτῳ) term to which A does not belong (1.19 82a11). Accordingly, if AeB and BaC are immediate propositions, then B is prior (προτέρῳ) to C among the terms to which A does not belong.\(^8\)

Just like immediate a-propositions, Aristotle takes immediate e-propositions to be indemonstrable principles of a science:

\[ \text{ὁμοίως δὲ καὶ εἰ τὸ Α τῷ Β μὴ υπάρχει, εἰ μὲν ἔστιν ἢ μέσον ἢ πρότερον ϕ οὐχ υπάρχει, ἔστιν ἀπὸδειξις, εἰ δὲ μή, οὐκ ἔστιν, ἄλλ’ ἀρχὴ, καὶ στοιχεῖα τοσαῦτ’ ἔστιν ὅσοι δροι· αἱ γὰρ τούτων προτάσεις ἀρχαὶ τῆς ἀπὸδειξίως εἰσιν. καὶ ωσπερ ἐνιαί ἀρχαὶ εἰσιν} \]

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\(^{8}\) *Post. An.* 1.19 82a9–13, 1.21 82b5–11.
ἀναπόδεικτοι, ὅτι ἐστὶ τόδε τὸδι καὶ ὑπάρχει τὸδε τῷδι, οὕτω καὶ ὅτι οὐκ ἔστι τὸδε τὸδι
οὐδὲ ὑπάρχει τὸδε τῷδι. (Post. An. 1.23 84b24–30)

Likewise, if A does not belong to B, then if there is a middle or prior term to which it
does not belong, there is a demonstration; and if not, there is not, but it is a principle.
And there are as many elements as terms; for the propositions consisting of these terms
are principles of the demonstration. And just as there are some indemonstrable
principles to the effect that this is this and this belongs to this, so too there are some to
the effect that this is not this and this does not belong to this.

According to this passage, immediate e-propositions are elements of the theorems
demonstrated from them. For example, if AeB and BaC are immediate propositions, they are
elements of the theorem AeC. This is illustrated by the following diagram, in which the
immediate e-proposition AeB is indicated by a zigzag line:

In this diagram, there is an e-path from A to C, composed of an atomic e-path from A to B
and an atomic a-path from B to C. The last two paths are proper parts of the e-path from A to
C. Hence, both AeB and BaC are prior in nature to AeC.

Similarly, if a theorem AeC is demonstrated from immediate propositions BaA and
BeC by means of Camestres, the e-path from A to C is composed of an atomic a-path from B
to A and an atomic e-path from B to C:
As before, this means that both BaA and BeC are prior in nature to AeC.

In the *Posterior Analytics*, Aristotle considers complex negative demonstrations in which a theorem is demonstrated by successive applications of Celarent, Cesare, Camestres, and Barbara. In these demonstrations, the conclusion corresponds to complex e-paths such as the following:

In this e-path, the terms B₃ and B₄ are disjoint in the sense that there is no a-path from the one to the other and there is no third term that can be reached from both terms by a-paths. More precisely, B₃ and B₄ are disjoint in the following sense:

**DEFINITION 3:** In any a-structure, two terms A and B are *disjoint* just in case

(i) A is distinct from B,

(ii) there is no a-path from A to B or vice versa, and

(iii) there is no term C in the a-structure such that there is an a-path from A to C and an a-path from B to C.

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89 *An. Post.* 1.21 82b4–21, 1.23 85a3–7, 1.25 86b10–27.
An e-path is a sequence of terms $A_1,A_2,\ldots,A_n$ in which one term, $A_i$, is disjoint from its successor, $A_{i+1}$. The term $A_{i+1}$ might be the endpoint of the e-path, $A_n$; but if it is not the endpoint, then there is an a-path from the former to the latter. Likewise, $A_i$ might be the starting point of the e-path, $A_1$; but if it is not the starting point, then there is an a-path from the former to the latter:

Thus, e-paths can be defined as follows:90

**DEFINITION 4:** Given an a-structure, an *e-path* is a sequence of terms $A_1,A_2,\ldots,A_n$ ($n > 1$) such that for some $A_i$ ($1 \leq i < n$):

(i) $A_i$ is disjoint from $A_{i+1}$,

(ii) either $i = 1$ or $A_i,A_{i-1},\ldots,A_1$ is an a-path, and

(iii) either $i+1 = n$ or $A_{i+1},A_{i+2},\ldots,A_n$ is an a-path.

90 Definition 4 could be strengthened by adding the condition that $A_i$ and $A_{i+1}$ be atomically disjoint (in the sense that there is no $B$ such that $A_i$ and $B$ are disjoint and there is an a-path from $B$ to $A_{i+1}$, and there is no $C$ such that $C$ and $A_{i+1}$ are disjoint and there is an a-path from $C$ to $A_i$). Since, however, this condition plays no role in the subsequent argument, I omit it (cf. n. 78).
With this characterization of e-paths in place, we are now in a position to give an account of priority in nature for e-propositions. As before, the account will appeal to the subpaths of an e-path. The subpaths of an a-path are exactly those a-paths that are proper parts of it:

**DEFINITION 5:** A subpath of an a-path $A_1, A_2, \ldots, A_n$ is any a-path $A_i, A_{i+1}, \ldots, A_j$ ($1 \leq i \leq n$) such that either $1 < i$ or $j < k$ (or both).

The subpaths of an e-path are all e-paths that are proper parts of it, and all subpaths of the two a-paths that may be contained in it:

**DEFINITION 6:** Let $A_1, A_2, \ldots, A_n$ be an e-path in which $A_i$ and $A_{i+1}$ are disjoint ($1 \leq i < n$). A subpath of this e-path is:

(i) any subpath of the a-path $A_i, A_{i-1}, \ldots, A_1$ (if $i \neq 1$),

(ii) any subpath of the a-path $A_{i+1}, A_{i+2}, \ldots, A_n$ (if $i+1 \neq n$), and

(iii) any e-path of the form $A_k, \ldots, A_i, A_{i+1}, \ldots, A_m$ ($1 \leq k \leq i$ and $i+1 \leq m \leq n$) such that either $1 < k$ or $m < n$ (or both).

An e-proposition $AeB$ is prior in nature to an e-proposition $CeD$ just in case some e-path from $A$ to $B$ is a subpath of some e-path from $C$ to $D$. Likewise, an a-proposition $AaB$ is prior in nature to an e-proposition $CeD$ just in case some a-path from $A$ to $B$ is a subpath of some e-path from $C$ to $D$. 

Demonstration by *reductio ad impossibile*
This account allows us to verify Aristotle’s claim in *Posterior Analytics* 1.26 that the premises of every direct demonstration are prior in nature to the conclusion. In particular, it allows us verify the claim that, whenever the premises of the moods Celarent, Cesare, and Camestres are scientific propositions, they are prior in nature to the conclusion. For example, consider an instance of Celarent in which the two premises, \( AeB \) and \( BaC \), are scientific propositions. The premises are either immediate propositions or demonstrable theorems in a given science. Hence, there is an e-path from A to B, and an a-path from B to C. The concatenation of these two paths is an e-path from A to C. The first two paths are subpaths of this e-path. Hence, \( AeB \) and \( BaC \) are prior in nature to \( AeC \).

Likewise, consider an instance of Camestres in which the two premises, \( BaA \) and \( BeC \), are scientific propositions. There is an a-path from B to A, and an e-path from B to C. The concatenation of these two paths is an e-path from A to C. The first two paths are subpaths of this e-path. Hence, \( BaA \) and \( BeC \) are prior in nature to \( AeC \). The same argument applies in the case of Cesare. Hence, given that the only moods that can be established by a direct deduction are Barbara, Celarent, Cesare, and Camestres, we have verified Aristotle’s claim that the premises of every direct demonstration are prior in nature to the conclusion.

By contrast, if the premises of a mood that relies on *reductio* are scientific propositions, it does not follow that they are prior in nature to the conclusion. Consider, for example, Aristotle’s demonstration by *reductio*, in which \( AoB \) is inferred from \( AeC \) and \( BaC \). If the two premises are scientific propositions, there is an e-path from A to C, and an a-path from B to C. These two paths, however, cannot be concatenated to form an e-path. Nor can they be concatenated to form any other path from A to B. As a result, the two paths are not subpaths...
of any path from A to B, and the premises AeC and BaC are not prior in nature to the conclusion, AoB.

In fact, as we have seen, Aristotle regards AeC as posterior in nature to AoB (1.26 87a27). In other words, AoB is prior in nature to AeC. Of course, Aristotle cannot mean that the particular negative proposition AoB can serve as a premise in a demonstration of AeC; for, in his syllogistic theory, no particular proposition can be used as a premise to deduce a universal proposition. Instead, AoB may be regarded as prior in nature to AeC on the grounds that AoB asserts part of content that is needed demonstrate AeC. On this account, AoB is prior in nature to AeC because the e-path from A to B is a subpath of the e-path from A to C. More generally, a scientific proposition is prior in nature to another scientific proposition just in case some path from the predicate to the subject of the former proposition is a subpath of some path from the predicate to the subject of the latter.

This characterization of priority in nature can extended to apply to all propositions that are satisfied in an a-structure. An a-proposition is satisfied in an a-structure just in case there is an a-path from the predicate to the subject term, and likewise for e-propositions. An i- or o-proposition is satisfied in an a-structure just in case its contradictory opposite, the corresponding universal proposition, is not satisfied in it:

91 See Pr. An. 1.24 41b22–7.
**DEFINITION 7:** For any a-structure and any terms A, B in this a-structure:

- AaB is satisfied in the a-structure iff there is an a-path from A to B
- AeB is satisfied in the a-structure iff there is an e-path from A to B
- AiB is satisfied in the a-structure iff there is no e-path from A to B
- AoB is satisfied in the a-structure iff there is no a-path from A to B

All the rules of Aristotle's assertoric syllogistic are sound with respect to this semantics:

whenever the premises of Aristotle's moods or conversion rules are satisfied in an a-structure, the conclusion is satisfied as well. In particular, this is true for the rule of a-conversion, according to which BiA may be inferred from AaB. This is because, as can be seen from the following theorem, a- and e-propositions are incompatible in a-structures:

**THEOREM 1:** In any a-structure, if there is an a-path from A to B, there is no e-path from A to B.\(^{92}\)

As mentioned above, any given science determines an a-structure. All scientific propositions of the form AaB, AeB, AiB, and AoB – both indemonstrable principles and the theorems demonstrated from them – are satisfied in the a-structure determined by the relevant science.

For any scientific propositions of the form AaB, AeB, AiB, and AoB, the relation of priority in nature can now be defined as follows:

**DEFINITION 8:** In any a-structure, a path is either an a-path or an e-path.

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\(^{92}\) This follows from Definition 3 and the fact that, if there is an e-path from A to B, then A and B are disjoint.
DEFINITION 9: For any propositions $AxB$ and $CyD$ that are satisfied in an a-structure (where ‘x’ and ‘y’ are placeholders for ‘a’, ‘e’, ‘i’, and ‘o’): $AxB$ is prior in nature to $CyD$ just in case some path from $A$ to $B$ is a subpath of some path from $C$ to $D$.

Just as before, the fact that a-structures are acyclic guarantees that this is a well-defined, asymmetric relation of priority in nature. This can be seen from the following theorem, which states that this relation is a strict partial order on the class of scientific propositions:

THEOREM 2: The relation of priority in nature introduced in Definition 9 is transitive and irreflexive. For transitivity, suppose that $AxB$ is prior in nature to $CyD$, and $CyD$ is prior in nature to $EzF$. Then some path, $P$, from $C$ to $D$ is a subpath of some path, $Q$, from $E$ to $F$. Moreover, some path from $A$ to $B$ is a subpath of some path, $P^*$, from $C$ to $D$. Now, let $Q^*$ be the sequence of terms that results from $Q$ by replacing the path $P$ with $P^*$. By Definition 8 and Theorem 1, either both $P$ and $P^*$ are a-paths or both of them are e-paths. Hence, the sequence $Q^*$ is a path from $E$ to $F$. Since some path from $A$ to $B$ is a subpath of $P^*$, it is also a subpath of $Q^*$. Hence, $AxB$ is prior in nature to $EzF$.

For irreflexivity, suppose that a proposition $AxB$ is prior in nature to itself. Then some path, $P$, from $A$ to $B$ is a subpath of some path, $Q$, from $A$ to $B$. First, suppose that $Q$ is an a-path. Then, by Definition 5, it follows that either $A$ or $B$ occurs twice on the a-path $Q$, contradicting the fact that a-structures are acyclic (Definition 1). Next, suppose that $Q$ is an e-path. By Theorem 1, it follows that $P$ is an e-path. Given Definition 6, this means that either $A$ or $B$ occurs twice on an a-path, contradicting the fact that a-structures are acyclic.
6. Accounting for Aristotle’s thesis in *Posterior Analytics* 1.26

As we have seen, the a-structures considered by Aristotle in the first book of the *Posterior Analytics* determine a relation of priority in nature among scientific propositions based on the a- and e-paths that obtain between the terms of the a-structure. This relation of priority in nature allows us to account for Aristotle’s thesis in *Posterior Analytics* 1.26 that direct negative demonstrations proceed from premises which are prior in nature to the conclusion, whereas demonstrations by *reductio ad impossibile* proceed from premises which are posterior in nature to the conclusion. The first part of the thesis, concerning direct demonstrations, is captured by the following theorem:

**Theorem 3:** For any instance of Barbara, Celarent, Cesare, and Camestres: if the premises are satisfied in an a-structure, then each premise is prior in nature to the conclusion.

The theorem holds because, in any a-structure, reasoning by Barbara amounts to extending a-paths, and reasoning by Celarent, Cesare, and Camestres amounts to extending e-paths. Since every scientific proposition is satisfied in an a-structure, it follows that any instance of these four moods in which both premises are scientific propositions proceeds from premises which are prior in nature to the conclusion.

Given that a ‘demonstration’ is any deduction in which the premises are scientific propositions, all demonstrations of the form Barbara, Celarent, Cesare, and Camestres proceed from premises which are prior in nature to the conclusion. Hence, in a deductive system in which these are the only moods in the three figures that do not rely on *reductio*,
every direct demonstration proceeds from premises which are prior in nature to the conclusion.

By contrast, this is not the case for demonstrations by *reductio ad impossibile*, such as those in which the premises and the conclusion constitute an instance of Felapton:

**THEOREM 4:** There are instances of Felapton in which both premises are satisfied in an a-structure and the conclusion is prior in nature to one of the premises.

This theorem can be established by means of the same arrangement of terms given by Aristotle in *Posterior Analytics* 1.26. Consider an a-structure which consists of the terms A, B, C, and a single a-path, from B to C:

![Diagram](image)

Both premises of Felapton, AeC and BaC, are satisfied in this a-structure. But the premise AeC is not prior in nature to the conclusion, AoB. On the contrary, since the e-path from A to B is a subpath of the e-path from A to C, the conclusion AoB is prior in nature to the premise AeC (see Definition 9).

Thus, there are instances of Felapton in which the conclusion is prior in nature to one of the premises. This is true not only for Felapton but for most of the particular moods of the assertoric syllogistic:
**THEOREM 5:** There are instances of Darii, Ferio, Festino, Darapti, Disamis, Datisi, Felapton, Ferison, and Bocardo in which both premises are satisfied in an a-structure and the conclusion is prior in nature to one of the premises.94

This theorem covers almost all of Aristotle’s standard particular moods in the three figures. The only exception is Baroco in the second figure: there are no instances of Baroco in which the premises are satisfied in an a-structure while the conclusion is prior in nature to one of the premises. For all other standard particular moods there are such instances, giving rise to demonstrations by *reductio* in which the conclusion is prior in nature to one of the premises.

This does not mean that there are no demonstrations by *reductio* in which both premises are prior in nature to the conclusion. There are such demonstrations by *reductio*.95

The point, though, is that demonstrations by *reductio*, unlike direct ones, do not guarantee the priority in nature of the premises over the conclusion. Thus, when Aristotle states that direct demonstration is better than demonstration by *reductio* on the grounds that the former proceeds from premises that are prior in nature to the conclusion whereas the latter proceeds

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94 For the negative particular moods (Ferio, Festino, Ferison, and Bocardo), this theorem can be established by means of the same a-structure given above in the proof of Theorem 4. For the affirmative particular moods (Darii, Darapti, Disamis, Datisi), the theorem can be established by means of an a-structure consisting of three terms, A, B, and C, connected by three a-paths: from A to B, from B to C, and from A to C. In this a-structure, AaC, BaC, AiC, CiB, and BiC are satisfied, but AiB is prior in nature to both AaC and AiC.

95 An example is a demonstration by *reductio* in which AiC is demonstrated from AaB and BiC, while both AaB and BaC are satisfied in an a-structure.
from premises which are posterior in nature to the conclusion (87a26–30), he does not mean that every instance of a direct demonstration is better in this respect than every instance of a demonstration by *reductio*. Rather, he means that the method of direct demonstration is superior to the method of indirect demonstration because the former but not the latter guarantees the priority in nature of the premises over the conclusion.\textsuperscript{96}

It is worth noting that the results stated in the above theorems are not unique to Aristotle’s theory of demonstration. There are analogous results, for example, in contemporary theories of grounding. These theories deal with the relation one truth being grounded in another, or one fact obtaining in virtue of another. This relation of grounding is usually taken to be connected in a systematic manner to the rules of inference posited in deductive systems. Consider, for instance, a deductive system for the language of propositional logic using the connectives of conjunction (∧) and negation (¬). The deductive system includes the following four direct rules of inference, where φ are ψ any propositions:

\begin{itemize}
  \item \textbf{Rule 1: Hypothetical Syllogism:}\[\phi \land \neg \psi \rightarrow \neg \phi\]
  \item \textbf{Rule 2: Disjunctive Syllogism:}\[\phi \lor \psi \land \neg \phi \rightarrow \psi\]
  \item \textbf{Rule 3: Addition:}\[\psi \rightarrow \phi \lor \psi\]
  \item \textbf{Rule 4: Simplification:}\[\phi \land \psi \rightarrow \psi\]
\end{itemize}

\textsuperscript{96} Similarly, Aristotle states that affirmative demonstration is better than negative demonstration on the grounds that the former is used in the latter but not vice versa (*An. Post*. 1.25 86b27–30). By this he does not mean that every negative demonstration makes use of an affirmative demonstration; for he countenances purely negative demonstrations that employ only negative moods (see 1.21 82b13–28 and 1.23 85a3–12). Instead, he means that there are negative demonstrations that make use of an affirmative demonstration to establish an affirmative proposition, whereas there is no affirmative demonstration that makes use of a negative demonstration (since negative propositions cannot appear in the derivation of an affirmative conclusion, 86b23–4).
φ, ψ, therefore φ∧ψ
¬φ, therefore ¬(φ∧ψ)
¬ψ, therefore ¬(φ∧ψ)
φ, therefore ¬¬φ

In addition, the system contains the following rule of *reductio ad impossibile*:

\[ \Gamma, \neg\varphi, \text{ therefore } \varphi \Rightarrow \psi \neg \neg \psi \]

Taken together, these rules suffice to derive all the laws of classical propositional logic. The four direct rules are special in that, when applied to true propositions, the conclusion is not only a logical consequence of the premises but is also grounded in these premises. For example, if φ and ψ are true, then φ∧ψ is grounded in these two propositions. The fact that φ∧ψ obtains in virtue of the fact that φ and the fact that ψ. More generally, the relation of ground is often taken to obey the following laws corresponding to the four direct rules of inference:

- If φ and ψ, then φ, ψ ground φ∧ψ
- If ¬φ, then ¬φ grounds ¬(φ∧ψ)
- If ¬ψ, then ¬ψ grounds ¬(φ∧ψ)
- If φ, then φ grounds ¬¬φ

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Thus, when the four direct rules of inference are applied to true propositions, the premises ground the conclusion. By contrast, this is not the case for derived rules that rely on the rule of *reductio ad impossibile*. For example, the above rule of *reductio* licenses the following derived rules in the system:

- $\varphi \land \psi$, therefore $\varphi$
- $\varphi \land \psi$, therefore $\psi$
- $\neg \neg \varphi$, therefore $\varphi$

When these rules are applied to a true proposition, this proposition does not ground the conclusion. For example, a conjunct of a true conjunction is typically not taken to be grounded in the conjunction. Thus, unlike deductions that instantiate the four direct rules, deductions using the rule of *reductio* may fail to reveal the grounds of the conclusion when applied to true premises.

The four direct rules of inference are analogous to the four moods that can be established by a direct deduction in Aristotle’s system. When the former are applied to true propositions, the premises ground the conclusion; when the latter are applied to scientific propositions, the premises are prior in nature to the conclusion. These rules and moods are

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98 In the first case, the premises are syntactically less complex than the conclusion (Correia 2014: 35 and 37). For example, both $\varphi$ and $\psi$ are syntactically less complex than $\varphi \land \psi$. In the second case, the premises are path-theoretically less complex than the conclusion, in the sense that every path for a premise is a subpath of some path for the conclusion. For example, if $AaB$ and $BaC$ are scientific propositions, then every path from $A$ to $B$ is
thus especially useful in contexts in which the focus is on deductions that proceed from what
grounds to what is grounded, or from what is prior in nature to what is posterior in nature.
However, these rules and moods do not suffice to generate all deductive consequences of a
given set of premises. To this end, one needs to employ the rule of *reductio ad impossibile* in
the two systems. Once this rule is admitted, deductions from true and scientific propositions
no longer follow the order of grounding or priority in nature.99

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a subpath of some path from A to C. Thus, in both cases, direct deductions proceed in the direction of increasing
complexity, either increasing syntactic complexity or increasing complexity of paths in an a-structure.

99 More specifically, the relation of priority in nature introduced in Definition 9 is comparable to the relation of
strict partial ground described by Fine (2012a: 71–3, 2012b: 7–9). Fine characterizes the latter relation in terms
of the idea of a fact verifying a proposition. On his account, if a truth φ is a strict partial ground of a truth ψ,
then there are truths φ₁, ..., φₙ such that, for any facts f, f₁, ..., fₙ: if f verifies φ and f verifies φᵢ for all 1 ≤ i ≤ n, then
the fusion of the facts f, f₁, ..., fₙ is a fact that verifies ψ. Thus, if φ is a strict partial ground of ψ, then every verifier
of φ is part of a verifier of ψ. Now, let us say that a scientific proposition AxB is *verified* by any path from A to B
in an a-structure. Then the characterization of priority in nature given in Definition 9 entails that, if a scientific
proposition AxB is prior in nature to a scientific proposition CyD, then there are scientific propositions φ₁, ..., φₙ
such that, for any paths P, P₁, ..., Pₙ: if P verifies AxB and Pᵢ verifies φᵢ for all 1 ≤ i ≤ n, then there exists a path
which is the concatenation of the paths P, P₁, ..., Pₙ and which verifies CyD. Thus, if AxB is prior in nature to
CyD, then every verifier of AxB is part of a verifier of CyD.
7. Parts and wholes

Aristotle’s argument from priority in nature relies on the claim that the e-premise of the direct negative demonstration, AeB, is prior in nature to the conclusion AeC (87a17–18). In support of this claim, Aristotle notes that the former proposition is a conclusion, while the latter is among those propositions ‘from which the conclusion derives’ (ἐξ ὧν τὸ συμπέρασμα):

πρότερα γάρ ἐστι τοῦ συμπεράσματος ἐξ ὧν τὸ συμπέρασμα· ἔστι δὲ τὸ μὲν Α τῷ Γ μὴ ὑπάρχειν συμπέρασμα, τὸ δὲ Α τῷ Β ἐξ οὗ τὸ συμπέρασμα. (An. Post. 1.26 87a18–20)

For the things from which a conclusion derives are prior to the conclusion; and that A does not belong to C is a conclusion, whereas that A does not belong to B is that from which the conclusion derives.

According to this passage, the propositions ‘from which a conclusion derives’ are prior in nature to the conclusion. Since AeC is a conclusion that derives from the premise AeB in Aristotle’s direct demonstration, the latter proposition is prior in nature to the former. Yet, this argument gives rise to an objection concerning Aristotle’s demonstration by reductio. For, in this demonstration, the conclusion AoB is derived from the premises AeC and BaC. Hence, if these premises are propositions ‘from which the conclusion derives’, it would seem that AeC is prior in nature to AoB, thus undermining Aristotle’s argument from priority in nature. Aristotle responds to this objection as follows:
Demonstration by reductio ad impossibile

οὐ γὰρ εἰ συμβαίνει ἀναφείσθαι τι, τούτο συμπέρασμα ἐστιν, ἐκείνα δὲ ἐξ ᾗν. ἀλλὰ τὸ
μὲν ἐξ ὧν συλλογισμὸς ἐστιν ὃ ἂν οὕτως ἔχῃ ὥστε ὁ δὲ πρὸς μέρος ἢ μέρος πρὸς
ὅλον ἔχειν, αἱ δὲ τὸ ΑΓ καὶ ΑΒ προτάσεις οὐκ ἔχουσιν οὕτω πρὸς ἀλλήλας. (Post. An.
1.26 87a20–25)

For it is not the case that, if something happens to be rejected, this is a conclusion and
the other things are that from which the conclusion derives. Rather, that from which a
deduction proceeds is what is related either as whole to part or as part to whole. But the
propositions AC and AB are not related to one another in this way.

In the first sentence of this passage, Aristotle deals with demonstrations by reductio, in which,
as he puts it, ‘something happens to be rejected’. The thing rejected in these demonstrations is
the assumption for reductio. Aristotle states that the proposition expressing this rejection is
not a ‘conclusion’ (συμπέρασμα). Thus, he denies that the proposition which is inferred in the
final step of the demonstration by reductio is a ‘conclusion’. Accordingly, Aristotle denies that
the premises employed in such a demonstration are ‘that from which’ this proposition
derives. With respect to Aristotle’s demonstration by reductio, this means that AoB is not a
conclusion, and that the premises AeC and BaC are not that from which AoB derives.

100 This is the text printed by Bekker (1831) and Waitz (1846). Ross (1949) replaces τὸ ΑΓ καὶ AB at 87a24 by τὸ
ΑΓ καὶ ΒΓ. The latter reading appears as a correction by a second hand in one MS (Vat. gr. 1024; see Waitz 1846:
43). All other MSS have AB instead of Ross’s ΒΓ. Ross (1949: ad loc) and Williams (1984: 64) claim that ΒΓ is
found in a secondary hand in MS Coisl. 330. However, this claim is not correct.

101 Aristotle holds that ‘a demonstration reducing to the impossible differs from an ostensive demonstration in
that it posits what it wishes to reject (ἀναφείσθαι) by reducing it to an agreed falsehood’ (Pr. An. 2.14 62b29–31).
See also the use of ἀναφείσθαι at SE 5 167b22–4.
In denying that AoB is a conclusion, Aristotle departs from his terminology in the Prior Analytics, where he refers to the proposition inferred in the final step of a deduction by *reductio* as a ‘conclusion’. In the passage just quoted, he adopts a stricter use of ‘conclusion’ on which only the propositions inferred in direct deductions count as ‘conclusions’. In a direct deduction, the conclusion ‘derives from’ the premises because the premises are related to one another ‘as whole to part or as part to whole’. In deductions by *reductio*, on the other hand, the premises are not related in this way. Thus, for example, while the premises AeB and BaC are related to one another ‘as whole to part or as part to whole’, this is not the case for the premises AeC and BaC.

In the last sentence of the passage, Aristotle states that ‘the propositions AC and AB are not related to one another in this way’. Presumably the point of this remark is not that these two propositions fail to constitute a pair of premises that are related to one another ‘as whole to part or as part to whole’; for, since both of these propositions are negative, they do not constitute a premise pair for any deduction. Rather, Aristotle’s point seems to be that the propositions AeC and AoB are not related in such a way that the latter can be derived from a AeC and another premise which relates to AeC ‘as whole to part or as part to whole’. Accordingly, AeC and AoB are not related in such a way that the latter is a conclusion and the former ‘that from which the conclusion derives’.

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102 Pr. An. 2.11 61a20, 61b17, 2.14 62b34, 62b38, 63b16.
In a number of passages throughout the *Analytics*, Aristotle claims that the premises of a deduction are related to one another ‘as whole to part’.103 He does not explain what it means for two premises to be related in this way. Nor is it clear whether, in these passages, he intends the claim to hold for all deductions in the three figures.104 In *Posterior Analytics* 1.26, at any rate, the claim seems to apply only to direct deductions but not to those by *reductio*. It is only in the former, but not in the latter, that the premises are related to one another ‘as whole to part or as part to whole’. Given the deductive framework employed by Aristotle in this chapter, this means that the claim applies to deductions of the form Barbara, Celarent, Cesare, and Camestres, but not to any other deductions in the three figures.

To see how the claim applies to direct deductions instantiating the four universal moods, consider the case of Barbara: AaB, BaC, therefore AaC. The major premise asserts

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104 Some commentators take the claim to apply to all deductions in the three figures (Barnes 1993: 189, Crivelli 2011: 169–70). However, it is not easy to see how the claim applies to deductions in the third figure. For example, how are the premises of a third-figure deduction in Darapti, AaB and CaB, related to one another as ‘part to whole’? Accordingly, Alexander takes Aristotle’s claim at *Prior Analytics* 1.25, 42a9–16, to apply only to deductions in the first figure (Alexander in *Pr. An.* 277.5–23). Similarly, Ross (1949: 379) maintains that the claim ‘is most clearly true’ of deductions in the first figure. In the same vein, von Kirchmann (1878: 116) takes the claim at *Post. An.* 1.26 87a20–25 to be restricted to first-figure deductions.
that, as Aristotle puts it, ‘A belongs to the whole of B’ (τὸ Α ὅλῳ τῷ Β ὑπάρχει).\(^{105}\) The minor premise asserts that ‘C is in B as in a whole’ (τὸ Γ ἐν ὅλῳ ἐστὶ τῷ Β).\(^{106}\) Thus, the major premise makes a universal claim about B as a whole, and the minor premise identifies a part of B that is in B as in a whole. In this sense, the major premise can be viewed as a ‘whole’ and the minor premise as a ‘part’. Similarly, in the case of Celarent, the major premise AeB asserts that ‘A does not belong to the whole of B’ (τὸ Α ὅλῳ τῷ Β οὐχ ὑπάρχει).\(^{107}\) Thus, the major premise makes a universal negative claim about B as a whole. Likewise, in the case of Cesare and Camestres, the e-premise asserts that the major or minor term does not belong to the middle term as a whole. Thus, in any deduction instantiating the four universal moods, one premise makes a universal affirmative or negative claim about the middle term, B, as a whole, while the other premise states that the minor term is a part of B which is in B as in a whole.

Apart from deductions of the form Barbara, Celarent, Cesare, and Camestres, there are no other deductions in the three figures of which this is true. It is thus plausible that, in Posterior Analytics 1.26, Aristotle characterizes direct deductions instantiating these four moods as those in which the premises are related to one another ‘as whole to part or as part to whole’.

In deductions of the form Barbara, Celarent, and Cesare, the major premise is related to the

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\(^{105}\) Pr. An. 2.2 53b30, 54b4–5, 54b25–6, 54b28–9, 55a6, 55a37, 2.3 55b27–8, 55b35–6, 56a26, 56a29–30, 56a33–4, 56b1, 2.4 56b38, 57a13–14, 57a19, 57a21, 2.21 67a33–4, 2.22 68a16, 68a22, 2.23 68b21, Post. An. 1.16 80a40–b1, 80b4, 80b8–9, 1.17 80b37–8.


\(^{107}\) Pr. An. 2.3 55b33, 2.4 56b38, 57a3, Post. An. 1.16 80b1, 80b8.
minor premise as whole to part, while in those of the form Camestres the major premise is related to the minor premise as part to whole.

When Aristotle employs the framework of his assertoric syllogistic in the first book of the *Posterior Analytics*, he focuses on the universal moods Barbara, Celarent, Cesare, and Camestres, but often does not mention any of the other moods in the three figures. Robin Smith takes this as an indication that the *Posterior Analytics* was largely composed by Aristotle before he developed the full syllogistic theory expounded in the *Prior Analytics*, and that ‘the *Posterior Analytics* as we have it does not presuppose the *Prior Analytics* but something decidedly simpler’. By contrast, the preceding considerations suggest another explanation of Aristotle’s focus in the *Posterior Analytics* on the four universal moods. These are the only moods that can be established by a direct deduction in the framework adopted by Aristotle in *Posterior Analytics* 1.26, and they are the only moods in which the premises are guaranteed to be prior in nature to the conclusion when applied to scientific propositions. As such, they are better suited for the purposes of scientific demonstration than the moods that rely on *reductio ad impossibile*. Thus, Aristotle may be focusing on these moods in the *Posterior Analytics* not because he was not sufficiently aware of the other moods when he wrote the treatise, but because these are the moods that are best suited for scientific demonstration.

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109 Smith 1982b: 327; see also Smith 1982a: 114–35.
Similarly, there is no reason to maintain, as Smith does, that *Posterior Analytics* 1.26 was written before *Prior Analytics* 2.11-14 because otherwise one ‘must suppose that Aristotle, after having acquired the sort of understanding of *reductio ad impossibile* reflected in *An. Pr.* 2.11–14, … somehow produced the very unsatisfactory treatment of this subject in *An. Post.* 1.26’. As we have seen, Aristotle’s argument in *Posterior Analytics* 1.26 is not inferior to or incompatible with the treatment of *reductio* elsewhere in the *Analytics*. While the argument is compressed, it is well suited to Aristotle’s purpose in the chapter and succeeds in showing that direct demonstrations but not those by *reductio* proceed from premises which are prior in nature to the conclusion.

Friedrich Solmsen has argued that the account of demonstration given by Aristotle in the first book of the *Posterior Analytics* is based on chains of terms arranged in order of increasing generality. Thus, as Robin Smith puts it, Aristotle’s ‘theory of demonstration is the theory of the structure of a system of terms arranged in ordered ‘chains’ (συστοιχίαι)’. In accordance with this view, I have argued that, for Aristotle, any given science determines an a-structure, in which terms are arranged in chains of immediate universal affirmations. Not only do these a-structures suffice to specify the truth-conditions of scientific a-, e-, i- and o-propositions. Given that these structures are acyclic, they give rise to a well-defined relation of priority in nature among scientific propositions. It is this relation of priority in nature, I

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110 Smith 1982a: 134


112 Smith 1982a: 122.
submit, that lies at the heart of Aristotle’s argument in *Posterior Analytics* 1.26 that direct demonstrations are better than those by *reductio ad impossibile*.

**References**


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